Bayesian Seismic Wavelet Extraction

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Outline of talk

• problem and wishlist
• solution
• example of multiple well wavelet extraction
• example of multiple offset AVO wavelet extraction
• uses of output
The forward problem: convolve well log posted in time with wavelet
The wishlist

- wavelet coefficients (plus uncertainty)
- wavelet length
- noise level (plus uncertainty)
- time to depth adjustments, that is checkshots (plus uncertainties)
  - DC shift
  - relative shift
- positioning adjustments (plus uncertainty)
- multi-well
- multi-stack
- deviated wells
Wavelet parameters

- Peak arrival time ($T_p$)
- Seismic samples from cubic spline
- Sparse model coefficients

Diagram shows a plot with time ($t$) on the x-axis and a wavelet shape on the y-axis, with key features labeled.
Checkshot parameters

Prior checkshot data with errors \( \{Z_i, \tau_i, \sigma_{\tau_i}\} \)

Conceivable time-to-depth map from prior
Registration parameters

seismic registration relative to well path

\[ \Delta t_R \]

\[ \Delta x_R, \Delta y_R \]
AVO parameters

\[ R = \frac{1}{2}(\Delta \rho / \rho + \Delta v_p / v_p) + B \theta^2 \left( \frac{1}{2} \Delta v_p / v_p - 2 v_s^2 (\Delta \rho / \rho + 2 \Delta v_s / v_s) / v_p^2 \right) \]

- normal incidence
- shear dependent
- anisotropy? background normal rotation?

\[ \theta^2 = v_p^2 / (V_{stack}^4 T_{stack}^2 / <X_{stack}^2>) \]

wavelet stretch allowed
Impedance blocking

- based on segmentation of p-wave impedance ($\rho v_p$)
  - maximum likelihood methods too expensive ( $O(N^2)$ )
  - Blended hierarchical stepwise segment/aggregate method ($O(N \log(k))$ )
- reduces noise
- increases speed
Likelihood function

\[
L(S, V_{\text{int}} | a_w, \tau, \Delta r_R, \sigma) = \prod_{\text{wells i stacks } j} N(S_{ij}(\Delta r_R) - w(a_w) \cdot r_{ij}(\tau), \sigma_j^2) \times \prod_{\text{wells i intervals } k} N(\Delta V_{\text{int}}(k), \sigma_v^2).
\]

Seismic data likelihood

- Synthetic seismic \( w(a_w) \cdot r_{ij}(\tau) \)
- Reflectivity \( r_{ij}(\tau) \) from linearised Zoeppritz
- Seismic data \( S_{ij}(\Delta r_R) \)
- Weighted subsampling of error to prevent "overcounting"

Interval velocity likelihood

- Sonic log velocity
- Checkshot interval velocity
- Backus-upscaled sonic-log velocity
- \( \Delta V_{\text{int}}(k) \)
Optimisation on wavelet length

Model selection – classic statistics problem

There exists sophisticated Bayesian model-selection procedures for general nonlinear regression problems. These will estimate the wavelet span distribution.

Newton or Ptolemy?
Optimisation and parameter uncertainty

Maximum Likelihood estimates

Quadratic approximation to uncertainties
(1) direct from finite difference Hessian or
(2) from Bayes' update formula
\[ C = (C_p^{-1} + X^T C_D^{-1} X)^{-1} \]

BFGS optimiser

BFGS methods: see Nocedal & Wright "Numerical Optimization"
\[ O(n^2) \times O(\text{forward model cost}) \times N(\text{models}) \]
Single well extraction

- **typical deterministic wavelet**
- **Bayes most likely wavelet**
- **seismic synthetic**
- **wavelet realizations**

- statistically insignificant
Multiple and deviated well extraction

deviated sidetrack

checkshot uncertainties go to prior with no logs

timed

seismic synthetic

original straight hole

looser checkshots with weaker amplitudes

timed

seismic synthetic

uncertainties go to prior with no logs

looser checkshots with weaker amplitudes

timed

seismic synthetic

TVD

timed

seismic synthetic

TVD

timed

seismic synthetic
Multiple offset extraction

Synthetic seismic AVO response

Stack angle

Near far

Decomposing wavelet span

Most likely model

Maximum likelihood wavelets with peak term in prior: registration corrects in time-to-depth map

Truth-case compared to inferred wavelet.
Uses of output

• Wavelet and noise level key input to:
  – Stochastic model based inversion
    – Glinsky et al., The Leading Edge 24, 990 (2005)
  – Multiple stack sparse spike inversion
    – Let It Wave (L’Ecole Polytechnique)

• Updated check shots and position used to post wells in interpretation systems
For more information on this algorithm

- [www.petroilume.csiro.au](http://www.petroilume.csiro.au) (open source)