# Bayesian Seismic Wavelet Extraction

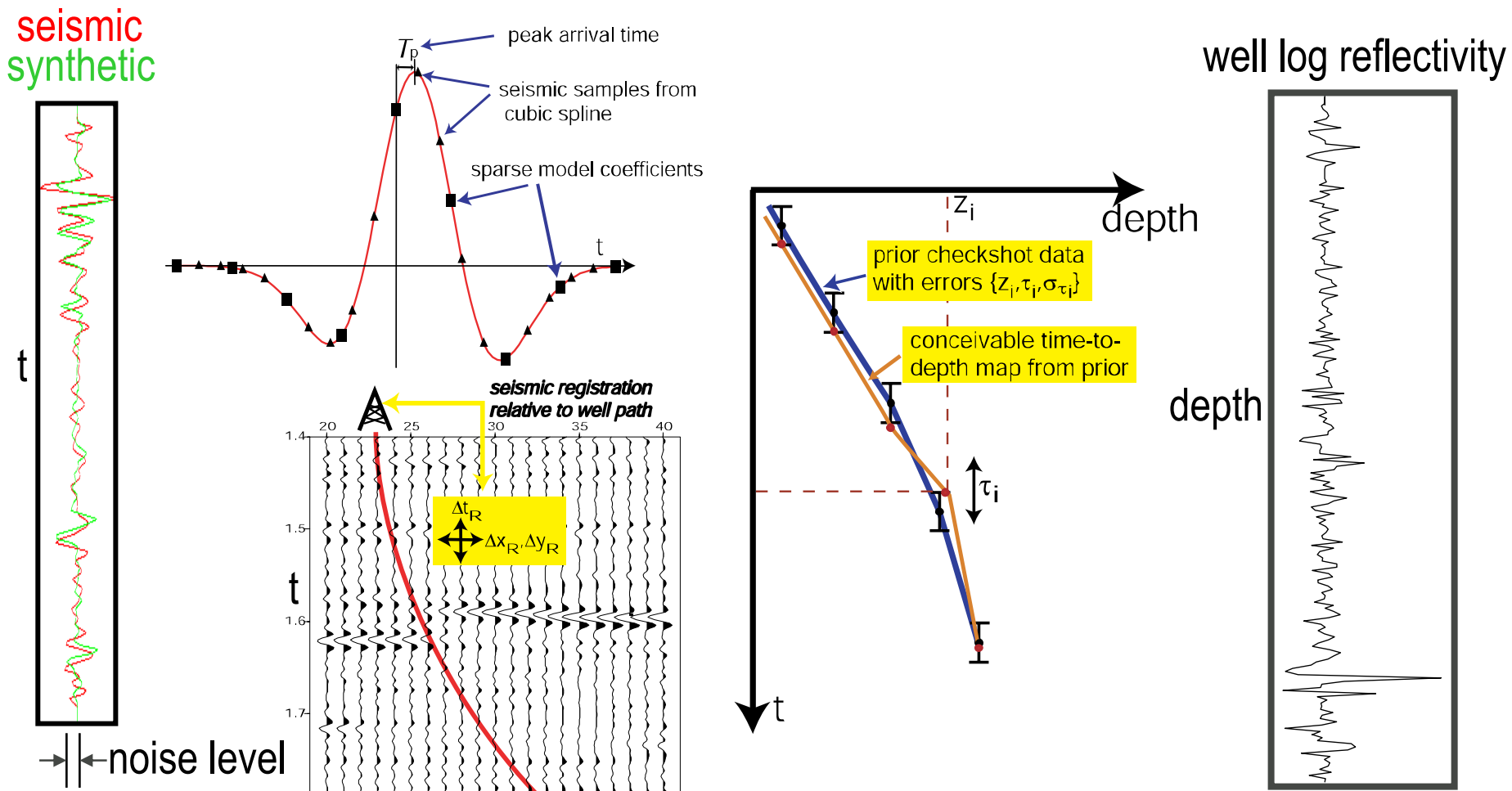
Michael Glinsky (BHP Billiton Petroleum, Houston)
James Gunning (CSIRO Petroleum, Melbourne)


www.oplnk.net/~glinsky

# Outline of talk

- problem and wishlist
- solution
- example of multiple well wavelet extraction
- example of multiple offset AVO wavelet extraction
- uses of output

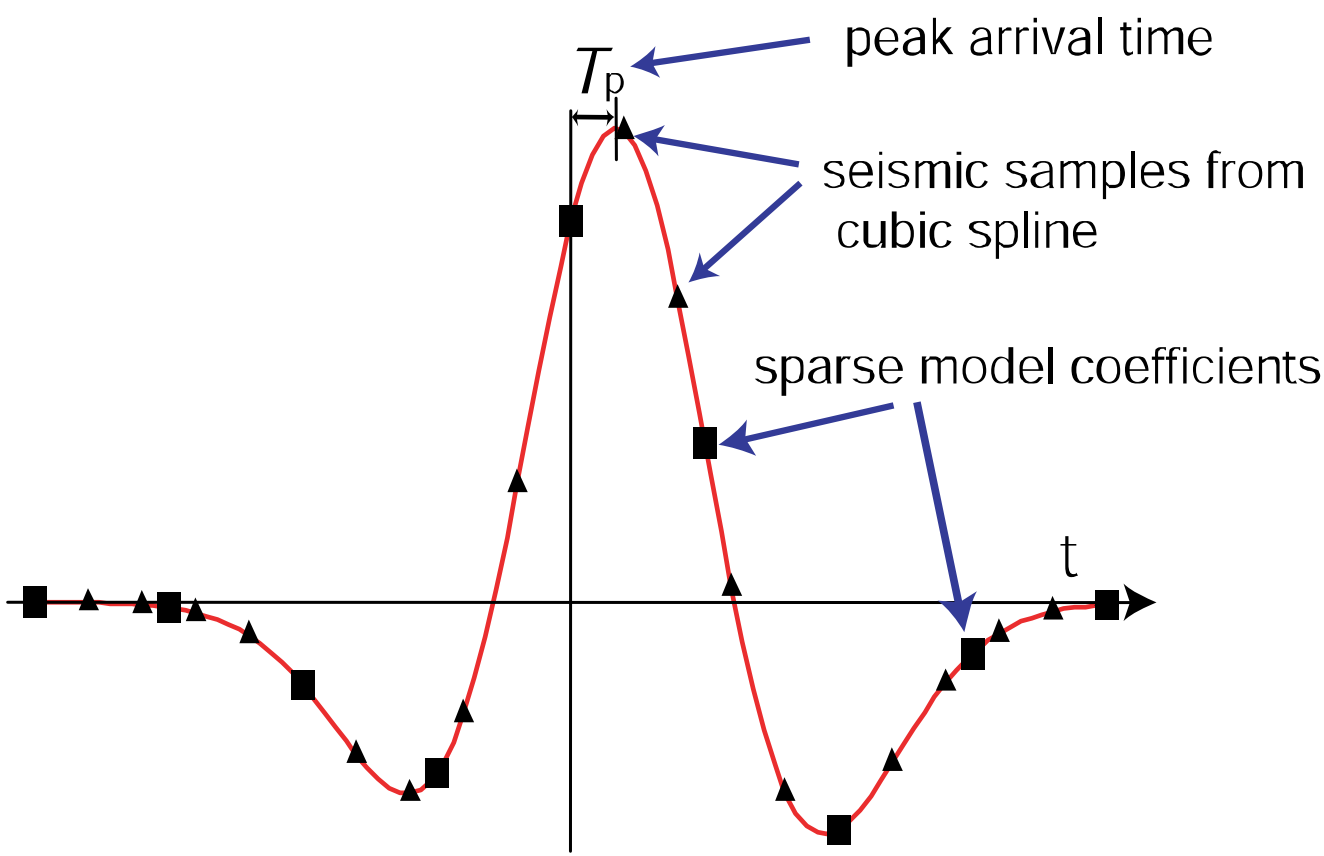
# The foreward problem: convolve well log posted in time with wavelet



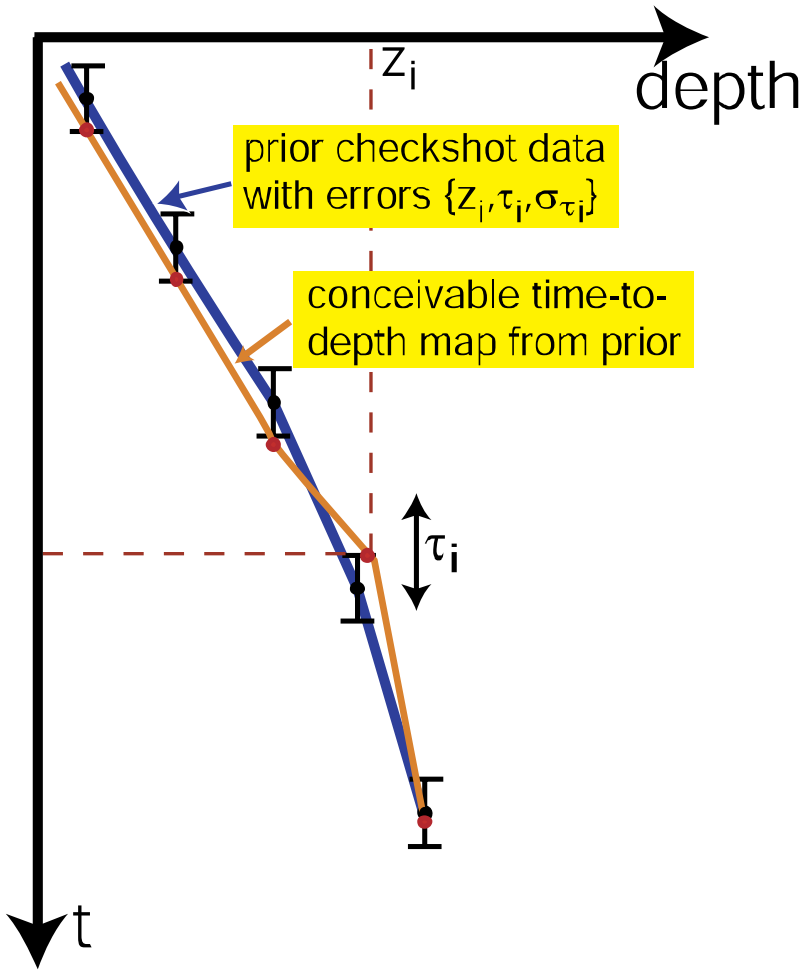
# The wishlist

- wavelet coefficients (plus uncertainty)
- wavelet length
- noise level (plus uncertainty)
- time to depth adjustments, that is checkshots (plus uncertainties)
  - DC shift
  - relative shift
- positioning adjustments (plus uncertainty)
- multi-well
- multi-stack
- deviated wells

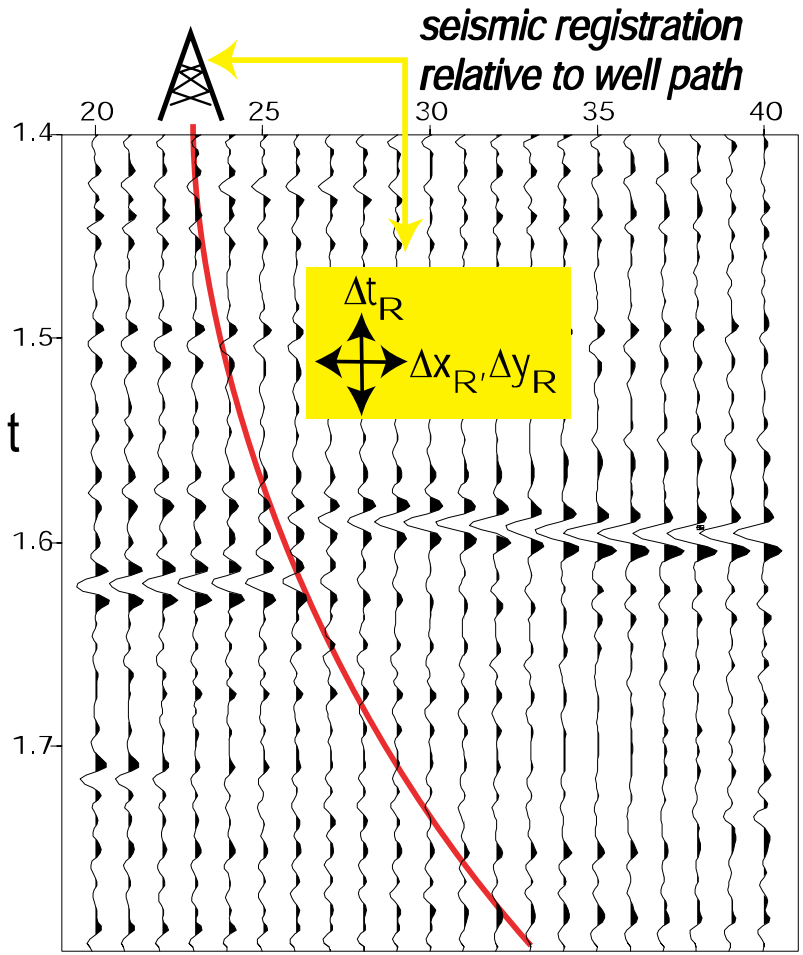
# Wavelet parameters



# Checkshot parameters



# Registration parameters



# AVO parameters

$$R = \tfrac{1}{2}\left(\frac{\Delta\rho}{\rho} + \frac{\Delta v_p}{v_p}\right) + B\,\theta^2\left(\tfrac{1}{2}\frac{\Delta v_p}{v_p} - 2\frac{v_s^2}{v_p^2}\left(\frac{\Delta\rho}{\rho} + 2\frac{\Delta v_s}{v_s}\right)\right)$$

normal incidence

shear dependent

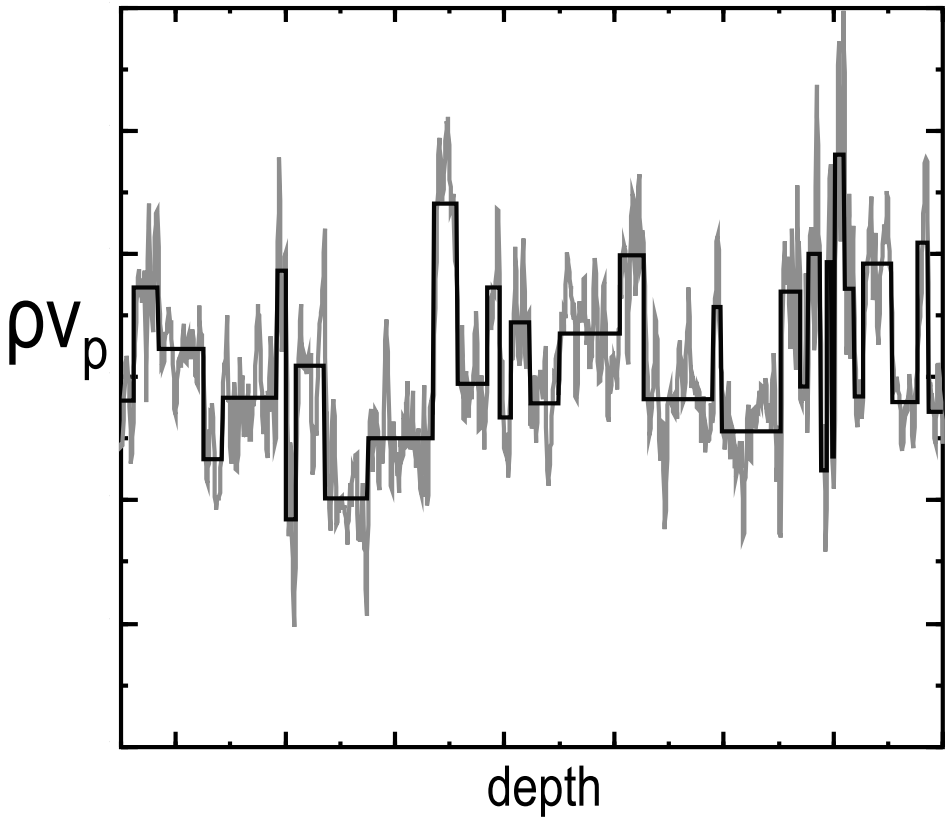
anisotropy? background normal rotation?

$$\theta^2 = v_p^2 / \left(V_{stack}^4 T_{stack}^2 / \langle X_{stack}^2 \rangle\right)$$

wavelet stretch allowed

# Impedance blocking

- based on segmentation of p-wave impedance ($\rho v_p$)
  - maximum likelihood methods too expensive ( $O(N^2)$ )
  - Blended hierarchical stepwise segment/aggregate method ($O(N \log(k))$ )
  - D.M.Hawkins, Comp. Stat. & Data Analysis 37(3), 2001
- reduces noise
- increases speed

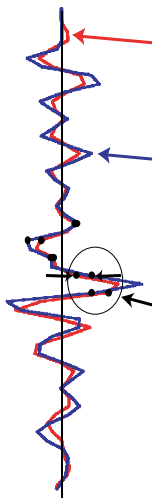


# Likelihood function

$$L(\mathbf{S},\mathbf{V}_{int}|\mathbf{a}_W,\tau,\Delta\mathbf{r}_R,\sigma) = \prod_{\text{wells i,stacks j}} N(\mathbf{S}_{ij}(\Delta\mathbf{r}_R)-w(\mathbf{a}_W)*\mathbf{r}_{ij}(\tau),\sigma_j^2)$$

$$\times \prod_{\text{wells i, intervals k}} N(\Delta V_{int(k)},\sigma_v^2).$$

## Seismic data likelihood

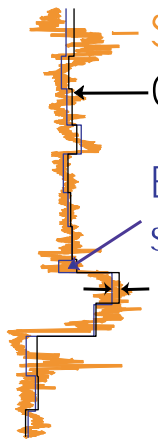


Synthetic seismic $w(\mathbf{a}_W)*\mathbf{r}_{ij}(\tau)$
Reflectivity $\mathbf{r}_{ij}(\tau)$ from linearised Zoeppritz

Seismic data $\mathbf{S}_{ij}(\Delta\mathbf{r}_R)$

Weighted subsampling of error
to prevent "overcounting"

## Interval velocity likelihood



Sonic log velocity

Checkshot interval velocity

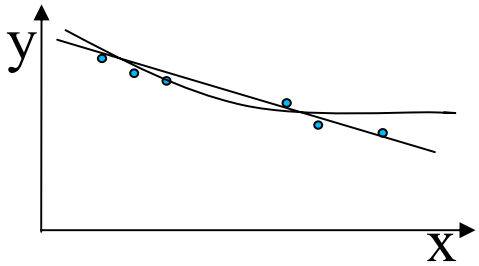
Backus-upscaled
sonic-log velocity

$\Delta v_{int(k)}$

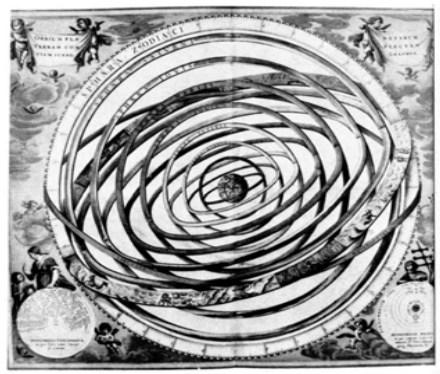
# Optimisation on wavelet length

**Model selection – classic statistics problem**

line ??, parabola ??, sextic ??




Newton or Ptolemy?

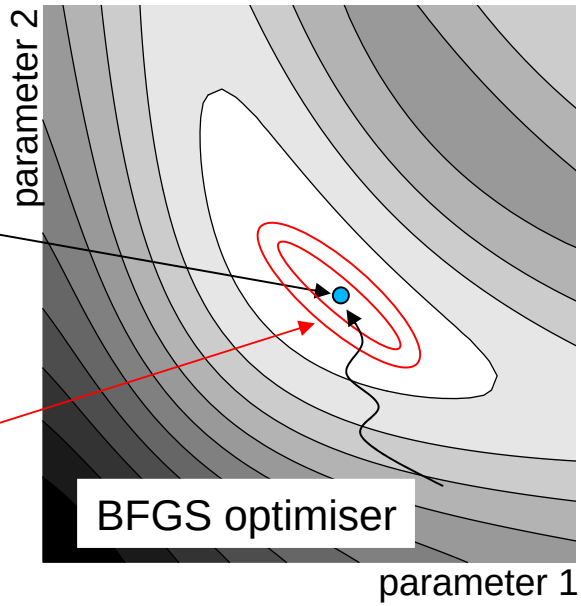


There exists sophisticated Bayesian model-selection procedures for general nonlinear regression problems. These will estimate the wavelet span distribution.

# Optimisation and parameter uncertainty

Maximum Likelihood estimates

Quadratic approximation to uncertainties
- (1) direct from finite difference Hessian or
- (2) from Bayes' update formula
- $C=(Cp^{-1}+X^{T}C_{D}^{-1}X)^{-1}$

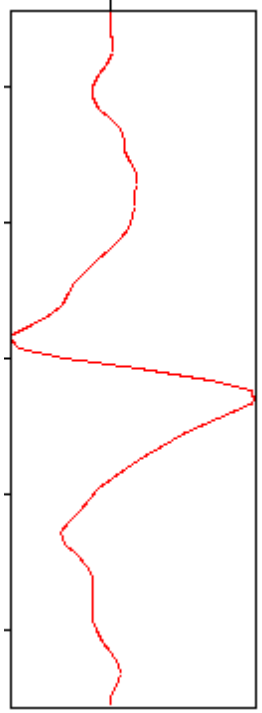



log(posterior) surface

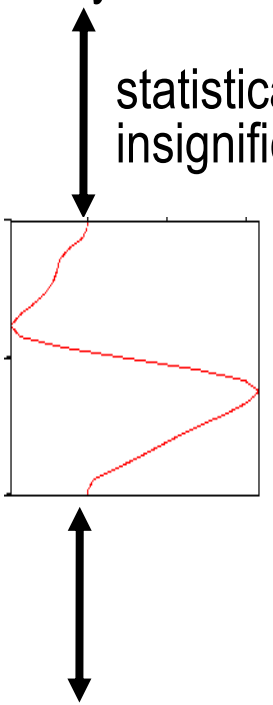
BFGS methods: see Nocedal & Wright "Numerical Optimization"
$O(n^2)$ x O(forward model cost) x N(models)

# Single well extraction

typical deterministic wavelet

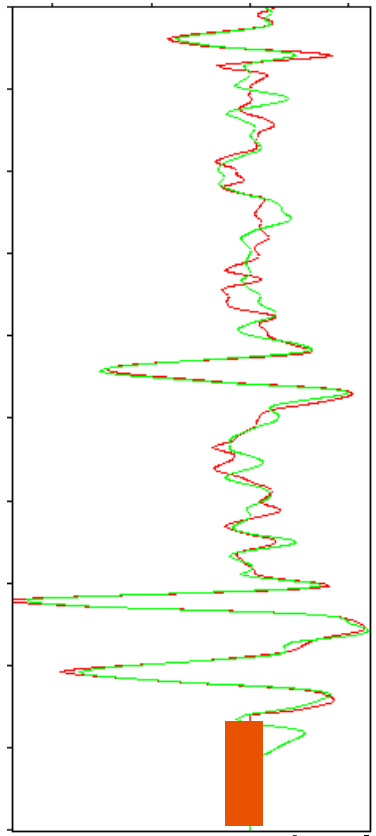


Bayes most likely wavelet

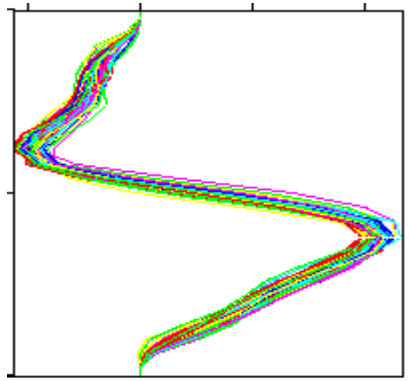


statistically insignificant

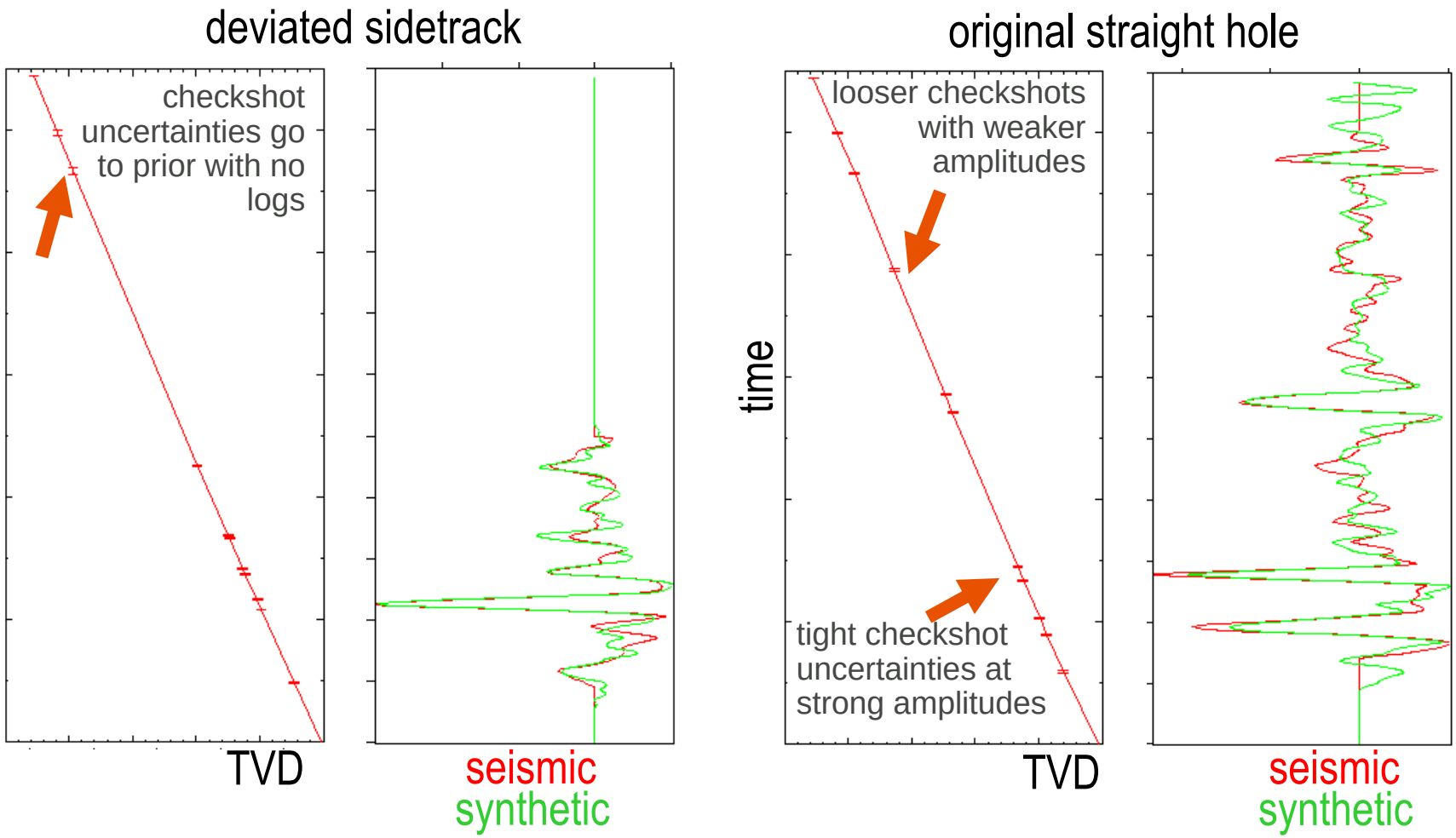
seismic
synthetic




wavelet realizations



# Multiple and deviated well extraction

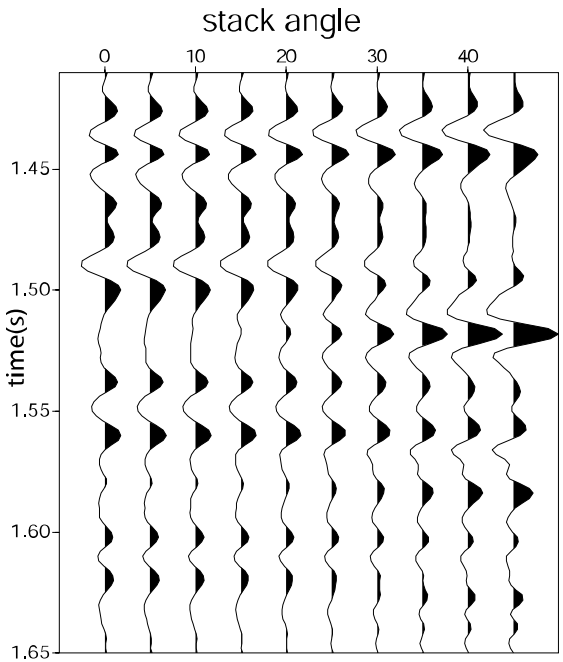




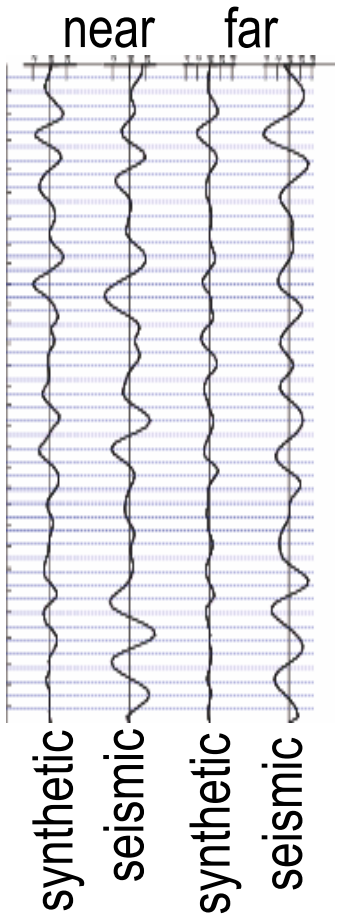

tvd

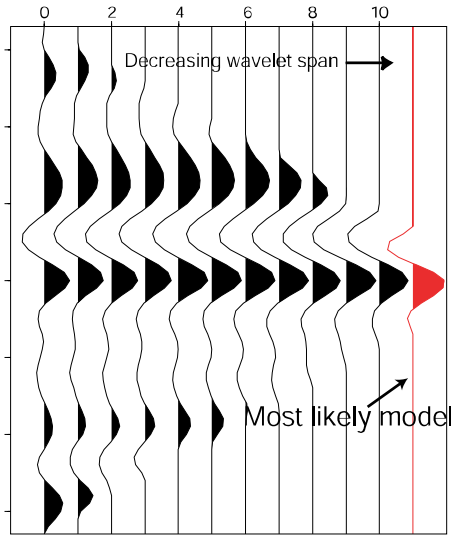
tvd

# Multiple offset extraction

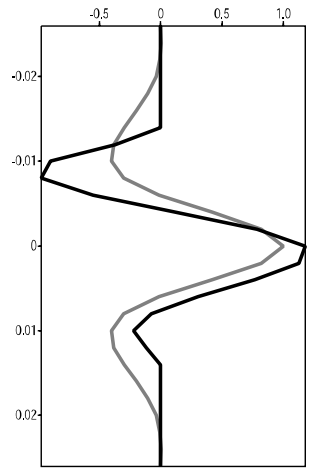



Synthetic seismic AVO response




Maximum likelihood wavelets with peak term in prior: registration corrects in time-to-depth map




Truth-case compared to inferred wavelet.

# Uses of output

- Wavelet and noise level key input to:
  - Stochastic model based inversion
    - Gunning and Glinsky, Computers and Geosciences 30, 619 (2004)
    - Glinsky et al., The Leading Edge 24, 990 (2005)
  - Multiple stack sparse spike inversion
    - Let It Wave (L'Ecole Polytechnique)
- Updated check shots and position used to post wells in interpretation systems

# For more information on this algorithm

- Gunning and Glinsky, Computers and Geosciences, *in press* (2006) www.oplnk.net/~glinsky
- www.petroleum.csiro.au (open source)