Consistent Downscaling of Seismic Inversions to Cornerpoint Flow Models
SPE 103268

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Contents

- Method overview and basics
- Handling pinchouts
- 2D examples
- 3D examples
- Conclusions
Overview

Seismic Inversion

Well ties, Velocity Analysis, Rock Models

Petrophysics, Log Interpretation

Geomodeling

Upscaling

Simulation

Gunning et al 2006
Integration of multi scale data

Sum of layer thicknesses simulated should approximately match seismic thickness
Methods
Bayesian Inference

\[ P(t | H, d_{\ell k}) = \frac{P(H | t, d_{\ell k}) P(t | d_{\ell k})}{P(H | d_{\ell k})} \]

- **Prior** from variogram and nearby data \( d_{\ell k} \)
- **Likelihood** from seismic mismatch
- Get the **posterior** by sampling many \( t \)
- **Normalizing** constant can be ignored
Truncated Gaussian Likelihood and Posterior

\[
p(t|d_{\ell k}) \sim N(\bar{t}, C_p)
\]

\[
p(H|t, d_{\ell k}) \sim N_k(\bar{H}, \sigma_H)
\]

\[
C_\pi = \left[ C_p^{-1} + TT^T/\sigma_H^2 \right]^{-1}
\]

\[
T_k = \begin{cases} 
0 & \text{if } t_k < 0 \\
1 & \text{otherwise}
\end{cases}
\]

"Stiff" nonlinear problem

\( t \) is a Gaussian proxy for \( h \)
Handling Pinchouts

- A Gaussian model is efficient and simple, but some of the proxies are negative
- For building geomodels set the thicknesses with negative proxies to zero
Truncated Gaussian Markov Chain Monte Carlo (TG-MCMC)

- Define auxiliary variable \( u_i = \{0, 1\} \) as indicator of truncation, 1 for \( t_i > 0 \)
  - Treats “configurational stiffness”
- Plausible truncations by Gibbs sampling
- Metropolis transition probability for \( \mathbf{t} \) includes thickness and auxiliary terms

\[
\alpha = \min \left( 1, \frac{\pi(t' | H, d_{\ell k}) \prod_{k=1}^{K} \pi(u_k | t_k')}{\pi(t | H, d_{\ell k}) \prod_{k=1}^{K} \pi(u_k | t_k)} \right)
\]

- Equivalent to sampling from the posterior
Assumptions and Performance

- Layer thicknesses are vertically uncorrelated at each trace
- Lateral correlations are identical for all layers
- Toeplitz form for resolution matrix

\[G = \begin{pmatrix}
\frac{1}{\sigma_t^2} + \frac{1}{\sigma_H^2} & \frac{1}{\sigma_H^2} & \dots & \frac{1}{\sigma_H^2} \\
\frac{1}{\sigma_H^2} & \frac{1}{\sigma_t^2} + \frac{1}{\sigma_H^2} & \dots & \frac{1}{\sigma_H^2} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{1}{\sigma_H^2} & \frac{1}{\sigma_H^2} & \dots & \frac{1}{\sigma_t^2} + \frac{1}{\sigma_H^2}
\end{pmatrix}\]

- Efficient Toeplitz solver
  - Handles layer drop-outs or drop-ins without refactoring
Sequential TG-MCMC

1. **Generate Path**

2. **Propose step $\Delta t$ from $N(0, C_{\pi})$ by $\Delta t = L \cdot \omega$ and accept $t' = t + \Delta t$ by Metropolis criterion**

3. **Krige means and variances for all layers at a trace**

4. **Propose $u_i'$ given $t_i$ using Gibbs**

5. **Draw a realization from the simulated posterior and add to the existing data**

6. **all traces**

7. **until convergence**

8. **Done**
2D Examples
A Simple Two Layer Case

\[ (\tilde{t}_2, \sigma_{\tilde{t}_2}) \]

\[ (\tilde{t}_1, \sigma_{\tilde{t}_1}) \]
A Simple Two layer Case
A Simple Two layer Case

- Bayes reconciles seismic and well/continuity data
  - Posterior covariance weights each data type appropriately
- Simulation retrieves the complete distribution, not just the most likely combination

\[
C_\pi = \left( C_p^{-1} + \frac{T T^T}{\sigma_H^2} \right)^{-1}
\]
Pinching Layer with Tight Sum Constraint

\[ \overline{H} = 4m, \, \sigma_H = 0.1m \]

\[ \overline{t} = (3m, 1m), \, \sigma_t = 1m \]
Prior sum not equal to Constraint

$H = 6m, \sigma_H = 0.5m$

$\bar{t} = (3m, 1m), \sigma_t = 0.5m$
3D Examples
3D Problem : Trends

(a) Trend in seismic thickness, $H$

(b) Trend in seismic noise $\sigma_H$; same $H$ trend as (a)

Simulations on a 100 x 100 x 10 cornerpoint grids with 25 conditioning data
3D Problem : Different Ranges

![Short Range](image1)

![Long Range](image2)

$$\overline{H} = 20 \text{m}, \sigma_H = 2 \text{m}$$
# Performance Summary

<table>
<thead>
<tr>
<th>Process</th>
<th>Work in Seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kriging Work</td>
<td>5.95</td>
</tr>
<tr>
<td>Toeplitz solver work</td>
<td>0.22</td>
</tr>
<tr>
<td>Overhead for all $10^4$ traces, 10 layers per trace</td>
<td>6.17</td>
</tr>
<tr>
<td>5000 samples, all traces</td>
<td>299.20</td>
</tr>
<tr>
<td>Total cost of simulation</td>
<td>305.37</td>
</tr>
</tbody>
</table>

*Using 2 GHz Pentium-M processor with 1 GB of RAM*

*Implemented in ANSI C, g77 compiler, using NR & LAPACK routines*

- 5000 samples for $10^5$ unknowns in 5min on a laptop
- 98% of computation is for generating and evaluating steps
  - Toeplitz solve is almost free
- Fewer samples could be used in practice
Conclusions

- TG-MCMC consistently downscales seismic inversions and integrates well and variogram data
- Auxiliary variables model truncated layers
- TG-MCMC is adequately efficient with Toeplitz assumptions
- Extensions for exact constraints and other properties seem feasible
Acknowledgements

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Delivery: Seismic Processing and Inversion Software

- **Bayesian preprocessing** (Gunning et al 2003, 2004)
  - Wavelet extraction
  - Time to depth maps
  - Well ties

- **Bayesian seismic inversion code** (Gunning et al 2005)
  - Set of plausible coarse scale reservoir models that honor seismic
  - Cornerpoint grid formats for reservoir simulation

- **Bayesian methods help integrate diverse, uncertain data**
Delivery Seismic Inversion

MCMC Samples from posterior distribution

\[ \pi(t, V_p, V_s, \phi, NG, \text{Fluid Type}, \ldots) \] for each layer
Truncated Gaussian Likelihood and Posterior

\[
p(t|d_{\ell k}) = \frac{1}{\sqrt{(2\pi)^K |C_p|}} \exp \left[ -\frac{1}{2} (t - \bar{t})^T C_p^{-1} (t - \bar{t}) \right]
\]

\[
p(H|t, d_{\ell k}) = \frac{1}{\sqrt{2\pi \sigma_H^2}} \exp \left[ -\frac{(t^T \tilde{T} - \bar{H})^2}{2\sigma_H^2} \right]
\]

\[
C_{\pi} = \left[ C_p^{-1} + T T^T / \sigma_H^2 \right]^{-1}
\]

\[
T_k = \begin{cases} 
0 & \text{if } t_k < 0 \\
1 & \text{otherwise}
\end{cases}
\]
Multi Facies Modeling

- Facies with different continuity can be sampled independently as there is no vertical correlation
  - need \((H_f, \sigma_{Hf})\) of individual facies
- Here two different facies are included
  - top 5 layers are highly continuous layers (large range)
  - bottom 5 layers have short range
Sampling when Seismic Constraint is Tight

- Only $K-1$ degrees of freedom are available as
  \[ \sum_{i=1}^{K} t_i = H \]

- Construct a new $K-1$ dimensional orthogonal basis using Gram-Schmidt or SVD

- Sample on this new basis $t'$

- Need to build (unique) transformation matrix $U$ mapping to original coordinates $t = U t'$
Ongoing Research

- Several Distinct Facies inclusion in each seismic loop
- Sampling on the constraint hyperplane
- Implementation of Block Methods to address the concerns with sequential methods
- Constraint on porosities and other nonlinear properties
- Selecting Realizations by upscaling the properties, simulating, and principle component analysis (PCA)
Markov chain Monte Carlo (MCMC)

- Samples from posterior using Markov and Monte Carlo properties.

- A Markov Chain is a stochastic process that generates random variables \( \{ X_1, X_2, \ldots, X_t \} \) where the distribution

\[ P(X_t \mid X_1, X_2, \ldots, X_{t-1}) = P(X_t \mid X_{t-1}) \]

i.e. the distribution of the next random variable depends only on the current random variable.

- These samples can be used to estimate summaries of the posterior, \( \pi \), e.g. its mean, variance.
Data Augmentation: Handles bends in the posterior

- Reversible MCMC hopping scheme that adjust to the proposals to the shape of local posterior
- Define auxiliary variable $u_i = \{0, 1\}$ as indicators of the layer occurrence
- Sampling in indicator space is done by Gibbs sampling
- This handles pinchouts; details of $t$ are handled in a Metropolis step
Metropolis for $t$

- It is possible to construct a Markov Chain that has the posterior as its stationary distribution.

- In the current step, the value of the parameters is $X_t$. Propose a new set of parameters, $Y$ in a symmetric manner.

- Calculate the prior and likelihood functions for the old and new parameter values. Set the parameter values in the next step of the chain, $X_{t+1}$ to $Y$ with probability $\alpha$, otherwise set to $X_t$.

$$\alpha(X, Y) = \min\left[1, \frac{\pi(Y)}{\pi(X)}\right]$$
Convergence of Mean and Variance

- Mean Convergence
- Std Deviation Convergence

- Should converge to target distribution in as few steps as possible
- Hopping
  - large steps $\rightarrow$ acceptance rate low
  - small steps $\rightarrow$ don’t explore posterior
  - Scaled posterior $\rightarrow$

$$\tilde{C} = \frac{5.67}{\tilde{C}}$$