Noise-thresholding sparse-spike inversion with global convergence: calibration and applications

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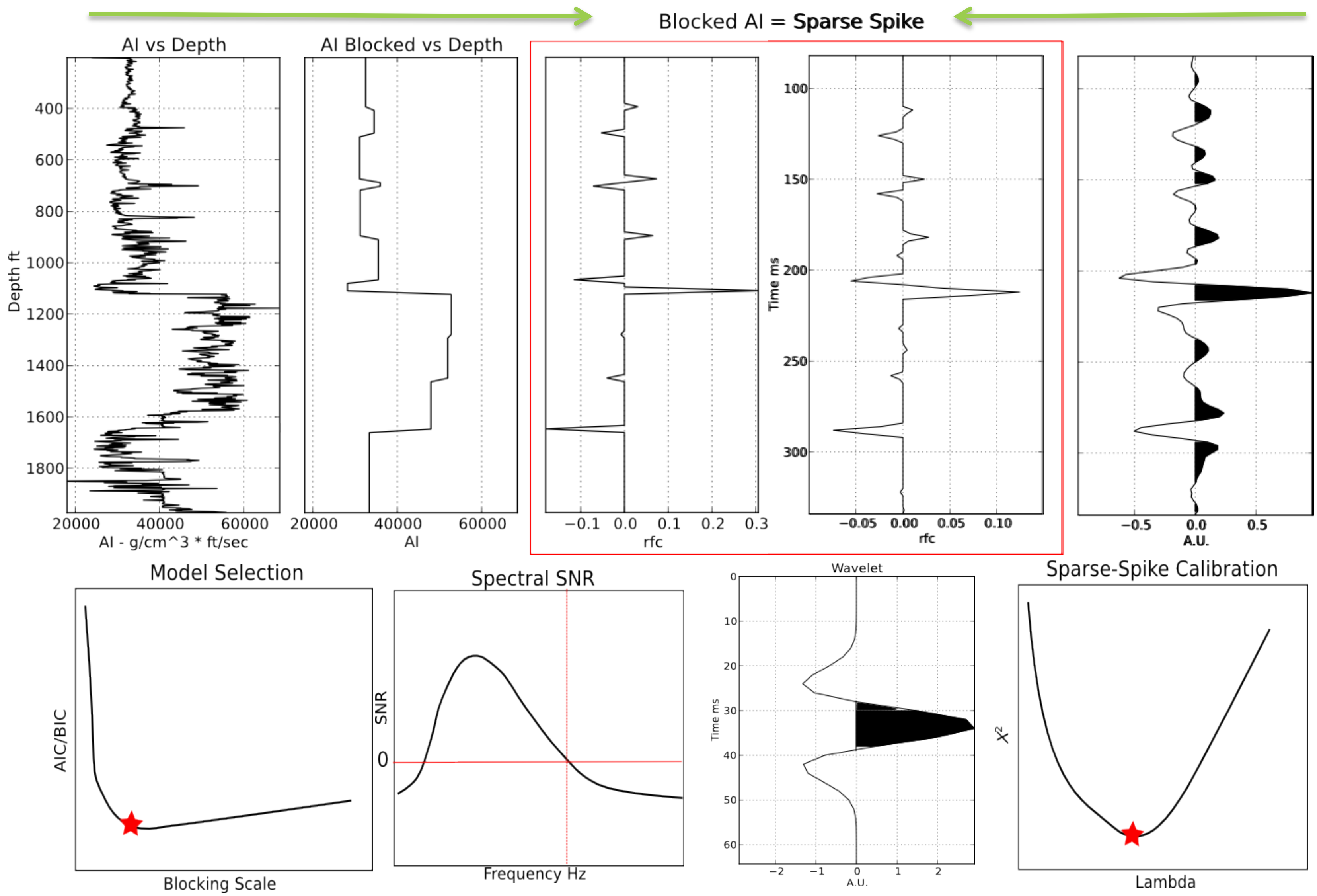
Illustrative Abstract

$$f = |W(t) * R_i(t) - S(t)|^2 + \lambda |R|^1$$

Wavelet Extraction & Model Selection

Sparse Spike Calibration & Inversion



Outline

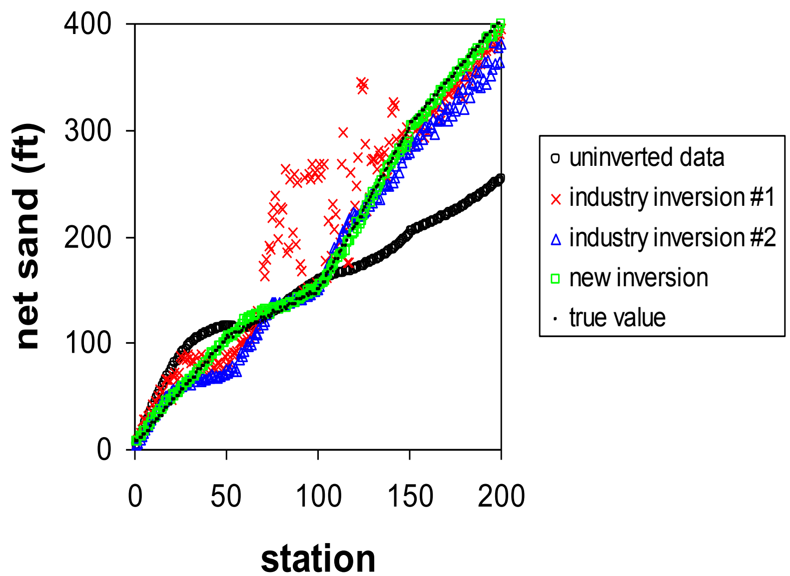
- Introduction – Why sparse-spike?
- Innovative Sparse-Spike Inversion
- Challenges to Calibration and Application
- Wavelet Extraction and Noise Estimation
- Model Selection – Scale Space Blocking
- Application

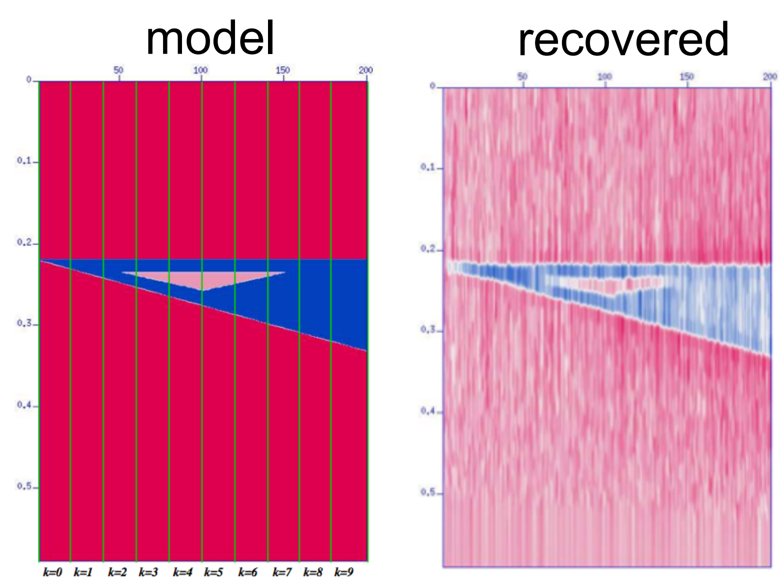
Why a sparse representation of the Earth?

- Discontinuities are commonly observed (changes in depositional style, lithology, unconformities….)
- Seismic reflections are a response to discontinuities (relative to seismic support scale) in impedance.

Sparse Spike Inversion

- Sparse-spike inversion removes the effects of the source wavelet from seismic data to reveal reflective interfaces, and is regularized (L1) such that it produces a **minimal set** of reflective interfaces.
- The sparse-spike inversion technique employed herein is based on the groundbreaking work of Daubechies (2003) - **proven convergence to norm**.





Noise-thresholding Sparse-Spike Inversion

- Debauchies provided a method for linear inverse problems where the solution has a sparse representation on a pre-assigned orthonormal basis.
- Amounts to repeated iterations of gradient descent and soft thresholding.

Minimize:

$$f = |W(t) * R_i(t) - S(t)|^2 + \lambda |R|^1$$

(L2 Attachment with L1 Regularization)

Algorithm:

1) L2 Optimization Step

$g_i = R_i + \mu\, W(\omega)^*(W(\omega) * R_i(\omega) - S(\omega))$, (Fourier Domain)

where $\mu = 1/\max\,(|W(\omega)|^2)$

2) Noise Thresholding – L1 Regularization

$F(g(t)) = \begin{cases} g(t) - T & if\ g(t) > T \\ g(t) + T & if\ g(t) < T \\ 0 & if\ |g(t)| < T \end{cases}$, where $T = \lambda\mu/2$ (Time Domain)

Rinse and repeat until convergence.

Limitations to Application of Noise Thresholding Sparse-Spike

- Method requires a good estimate of the wavelet.
- Requires a good estimate of the noise.
  - The trade-off term (λ) that dictates the sparsity must be calibrated.

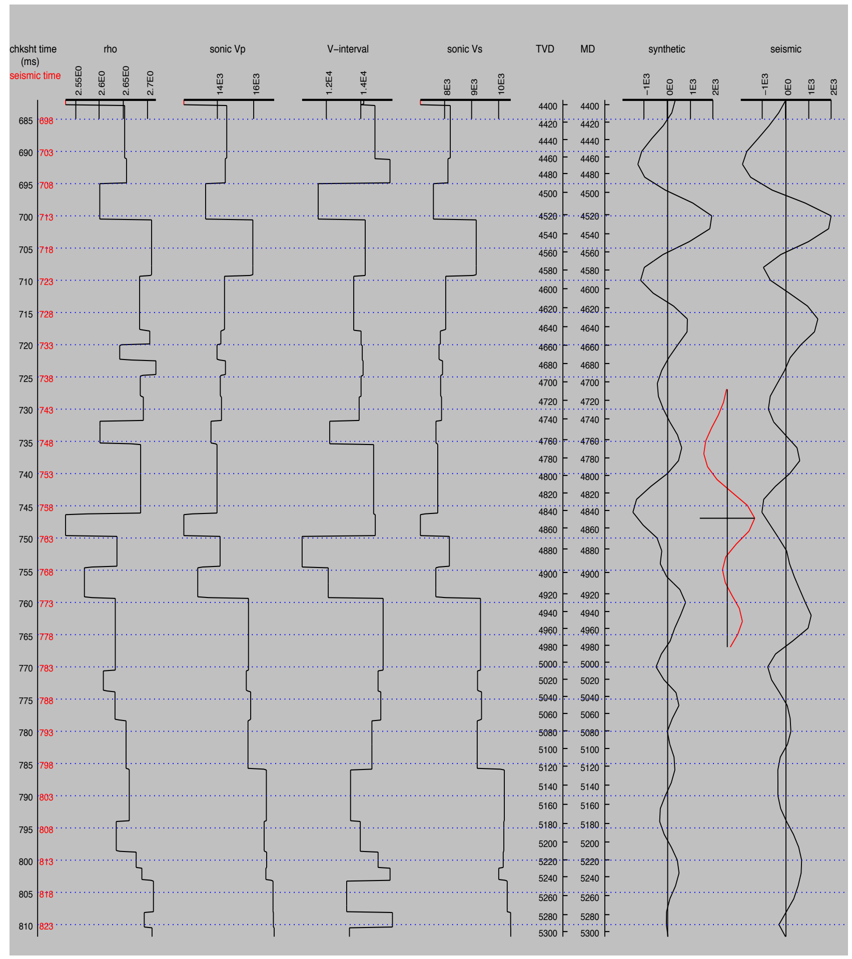
$$f = |W(t) * R_i(t) - S(t)|^2 + \lambda|R|^1$$

Bayesian Wavelet Estimation

Delivery (CSIRO) waveletExtractor

- Uses seismic data, **logs** and checkshots (single or multiple wells).
- Searches for the most-likely wavelet within a range of possible:
  - Registrations (time-depth model)
  - Well-positions/deviations
  - Wavelets
  - **Noise**
- Reports the most-likely model with uncertainty.

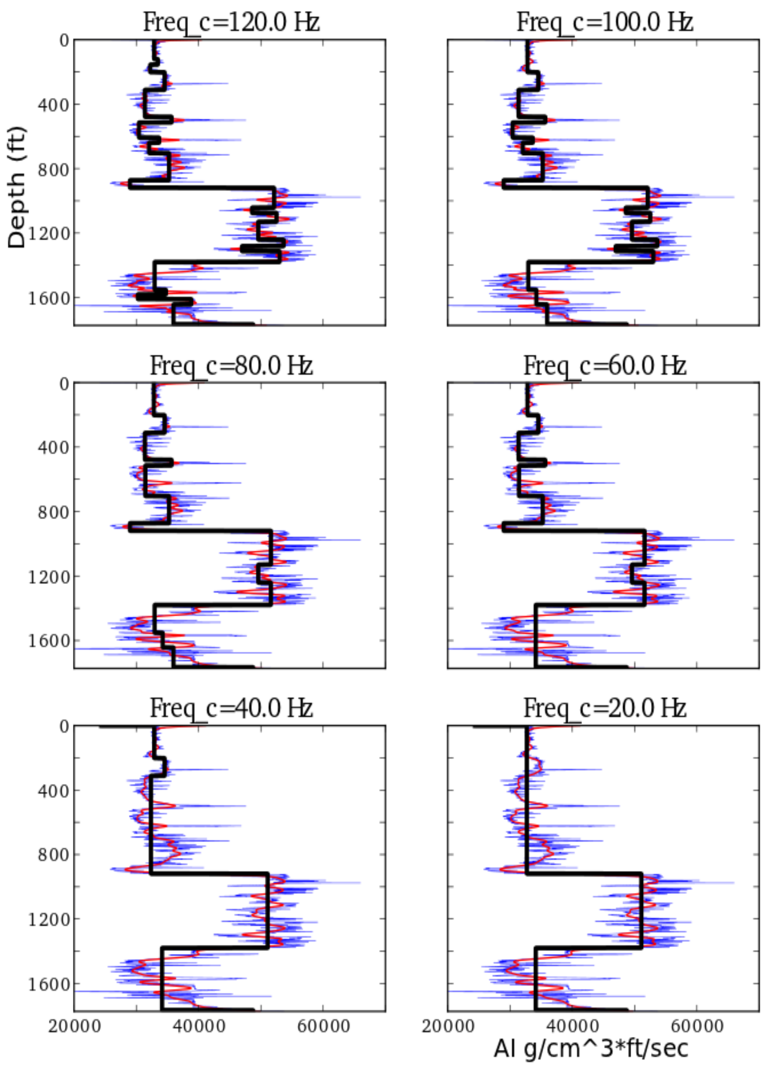
Matching Synthetic to Seismic



Blocking the Model: *Model Selection*

Motivation:

- We would like a model of **sufficient complexity** to adequately describes observed data, but not so complex so that we fit noise, waste resources...
- Selecting the appropriate model complexity provides the insight into how to optimize regularization of our sparse-spike inversion.

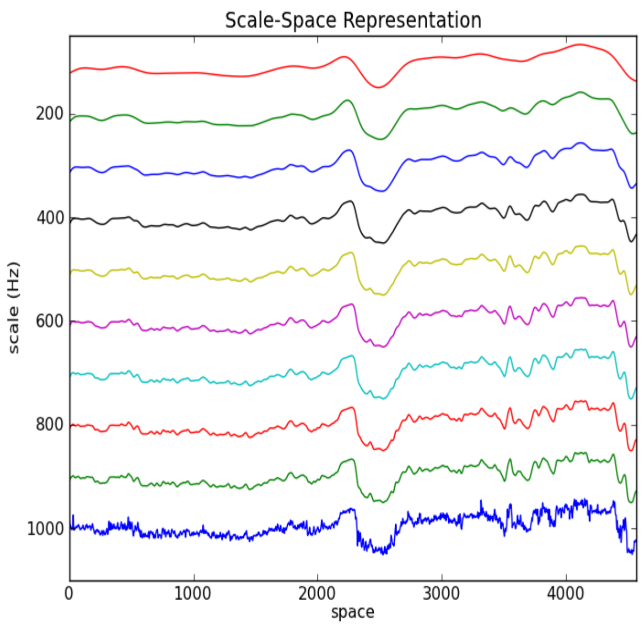


Blue – data, Red – filtered data, **Black - Segmentation**

Scale-Space Segmentation

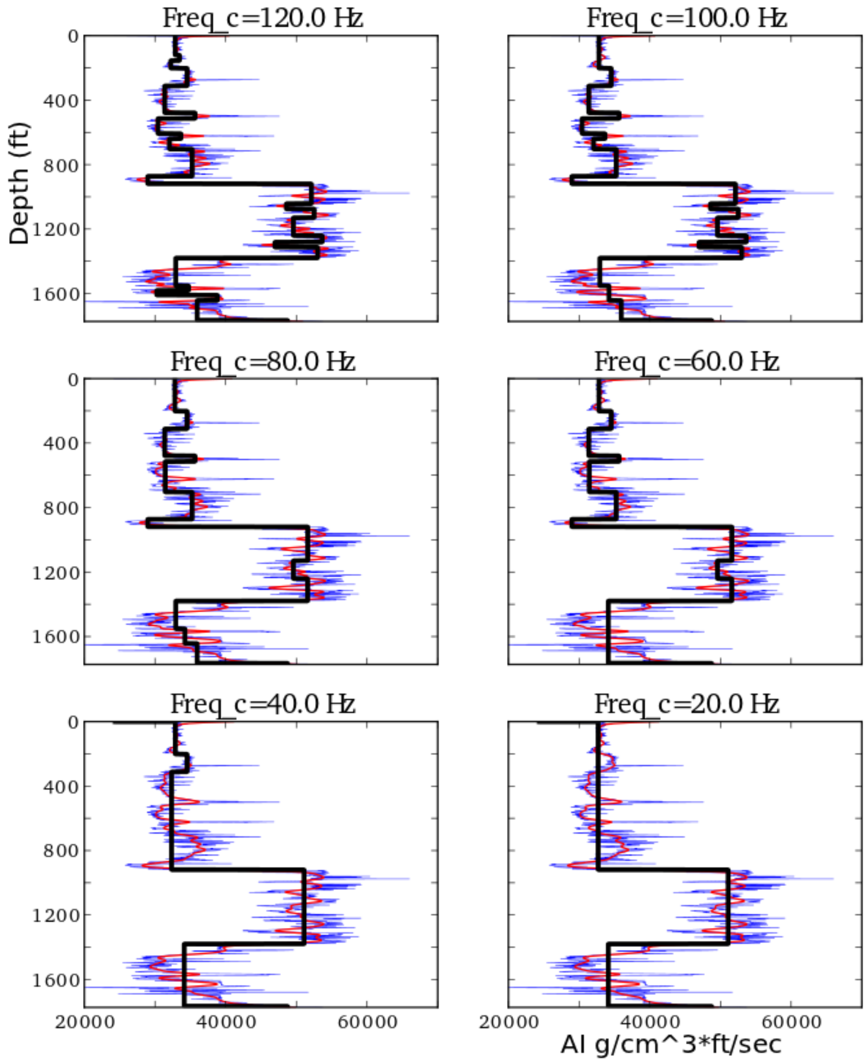
$$F(z,\sigma) = f(z) * g(z,\sigma) = \int_{-\infty}^{\infty} f(u) \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-u)^2}{2\sigma^2}} du$$

$$\sigma = \frac{1}{2\pi f_c} \qquad \frac{\partial^n F}{\partial z^n} = f * \frac{\partial^n g}{\partial z^n}$$



- Scale-Space segmentation is an edge detection method often used in image processing and computer vision.
- A scale-space is made by convolutions of successively larger support-scale (σ) Gaussian wavelets with the data.
- The 1st derivative provides the modulus, and the zero crossing-points of the 2nd derivative provides the inflection points.

Scale-Space Segmentation: Seismic Analogy

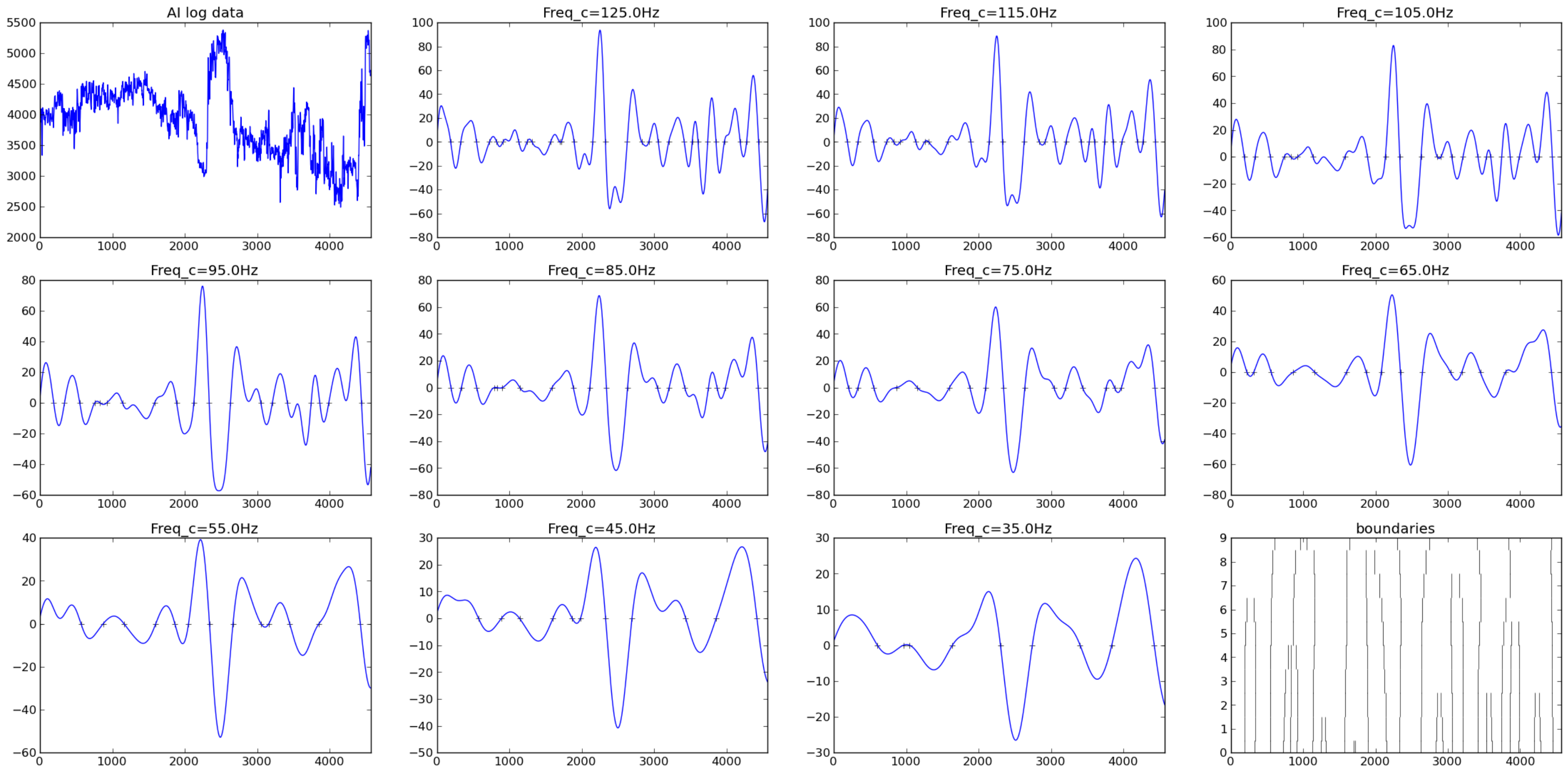


- Analogous to seismic convolution model:
- Input: *ln*(Impedance)
- the 1st derivative of the scale-space filtered impedance gives linearized reflectivity
- The 2nd derivative corresponds to the maxima of the salient reflections at that scale

Blue – data, Red – filtered data, **Black - Segmentation**

Segmentation without Localization

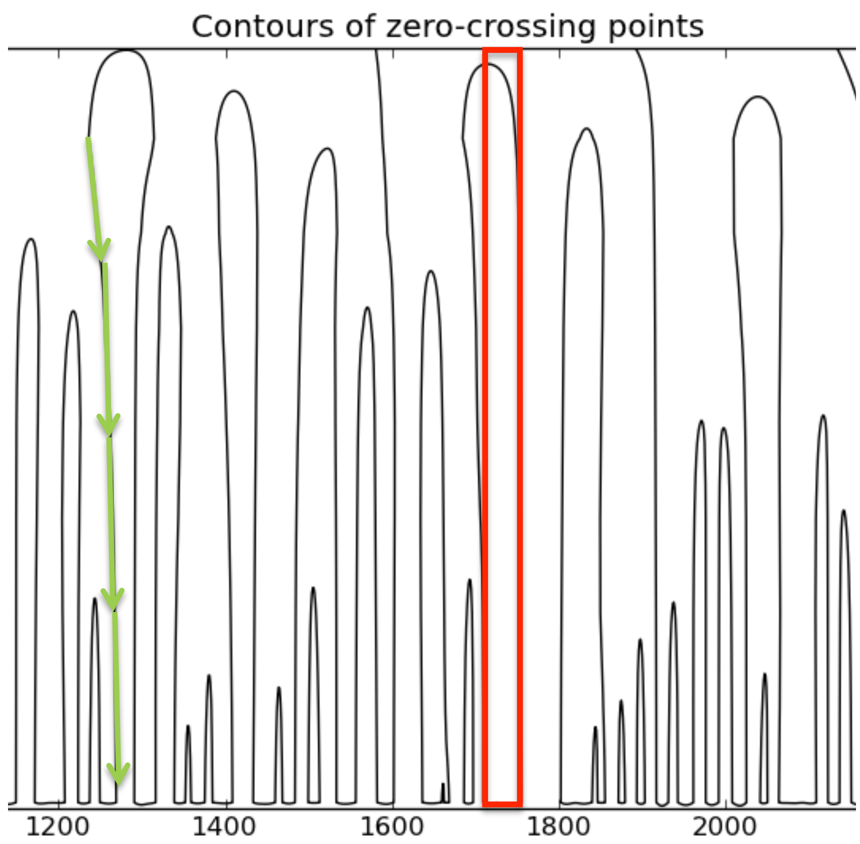
2nd Derivative of Scaled Data



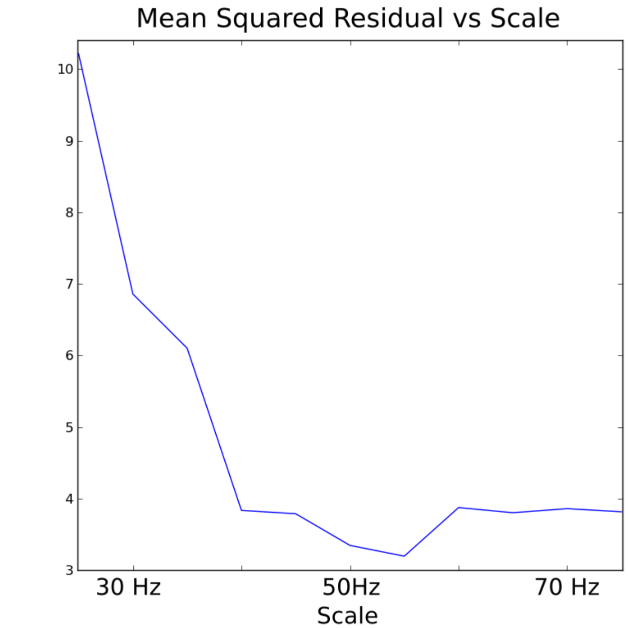
Cut of frequency is the frequency at which the amplitude is cut in half.

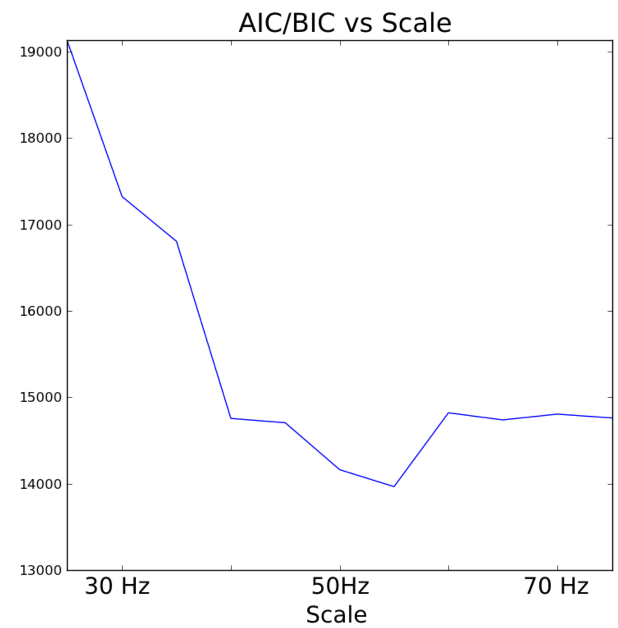
Localization

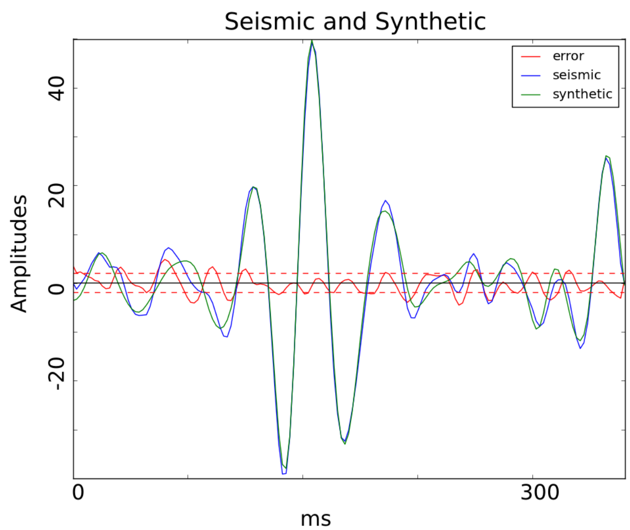
- As the support scale of the scale-space becomes larger, the location of the inflection points drift.
- Localization is the procedure of placing the inflection points back to the high resolution points.

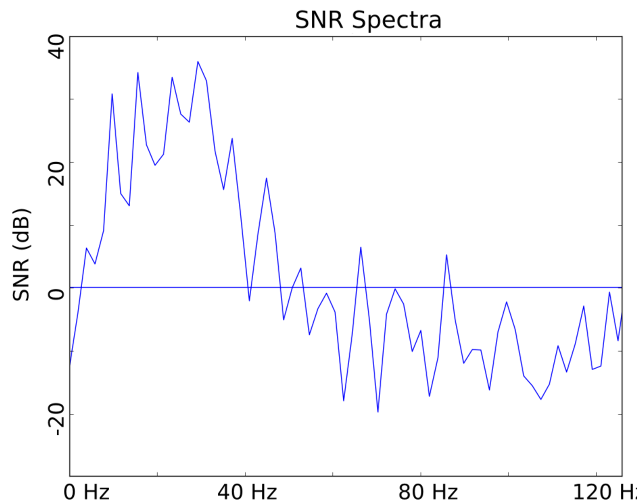


Selecting the Appropriate Support Scale





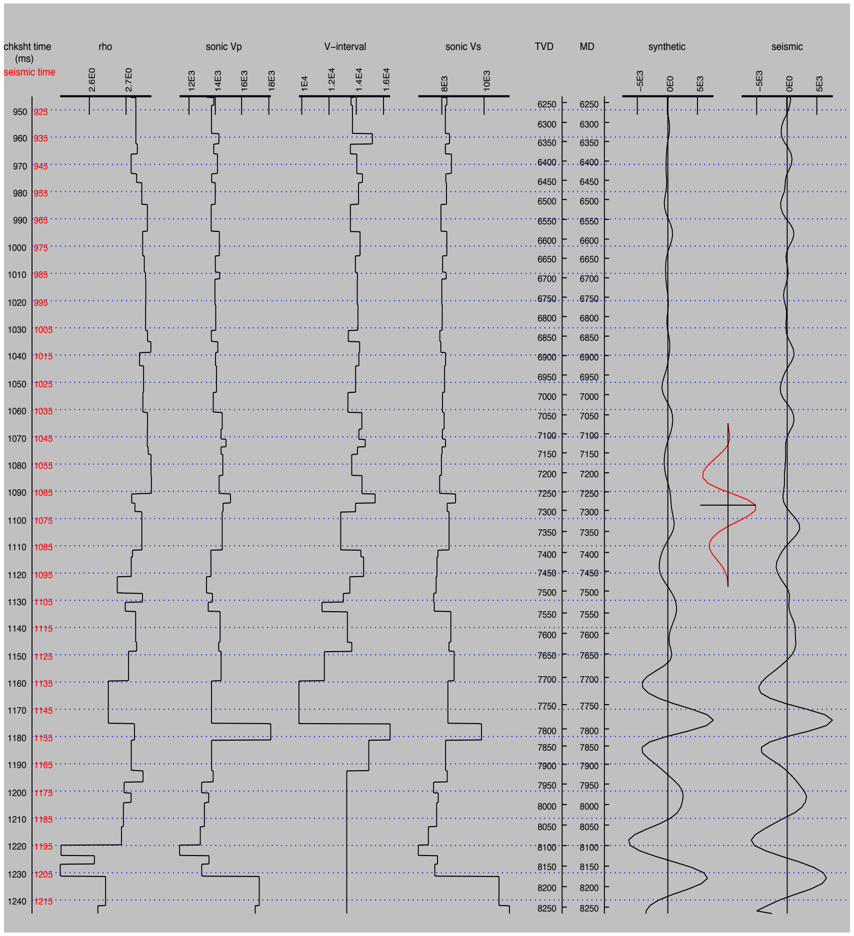


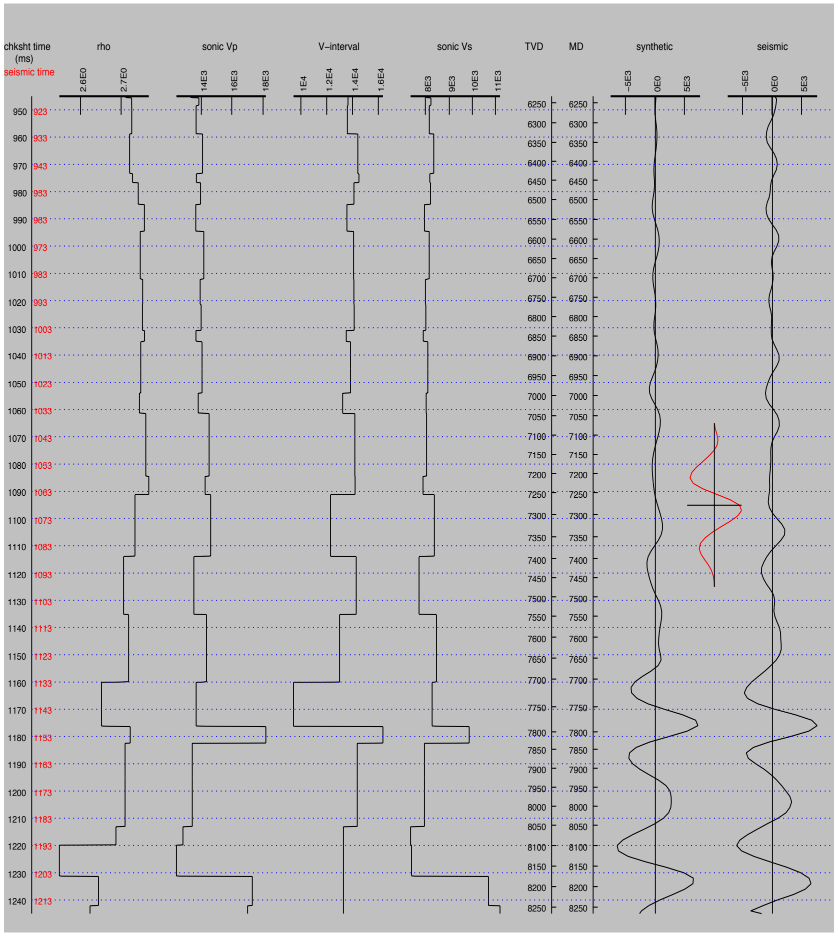


- **The appropriate support scale is dictated by the bandwidth of the signal above noise.**
- Too much smoothing loses blocks necessary for fitting the model.
- Too little smoothing provides the freedom to fit noise.

Example of Blocking Scale

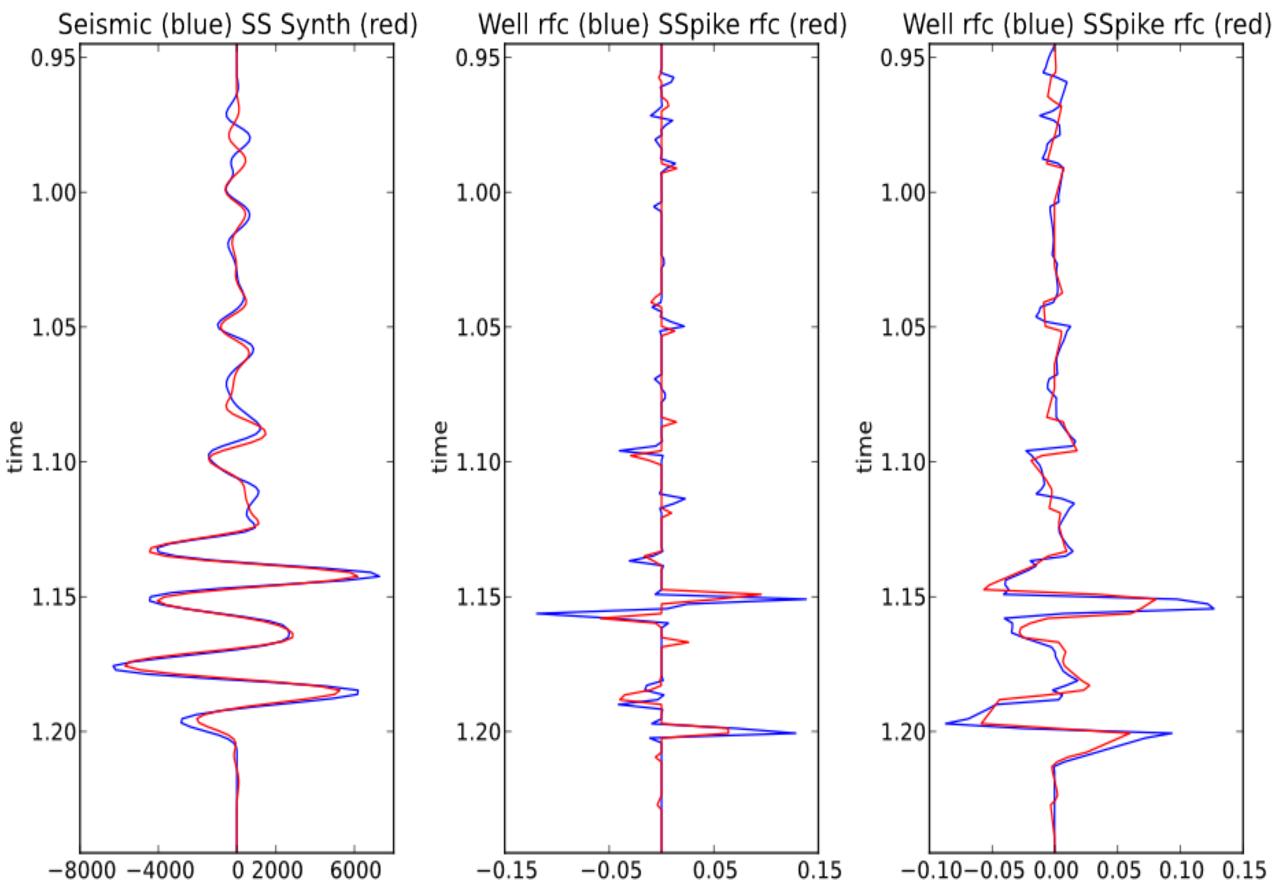
Density Vp Vint Vs Synthetic/Seismic Density Vp Vint Vs Synthetic/Seismic





125 Hz Scale R=0.965, RMS noise=557. **65 Hz Scale** R=0.967, RMS noise=558.

Calibration of Sparse-Spike



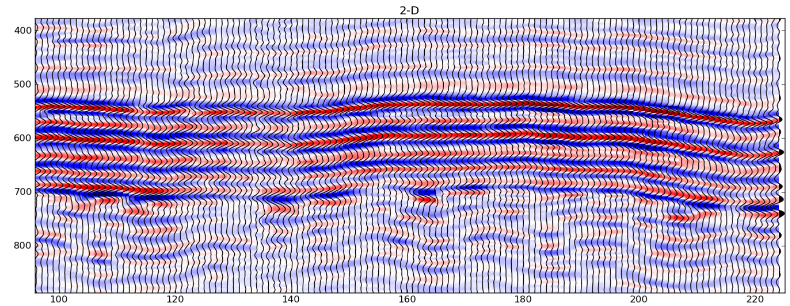
Important Links:

- **Bandwidth Above Noise ~ Model Complexity ~ λ**
- **Match between reflectivity and sparse-spike: $\lambda \approx \mu_{noise}$**

$$f = |W(t) * R_i(t) - S(t)|^2 + \lambda |R|^1$$

Impedance Profile

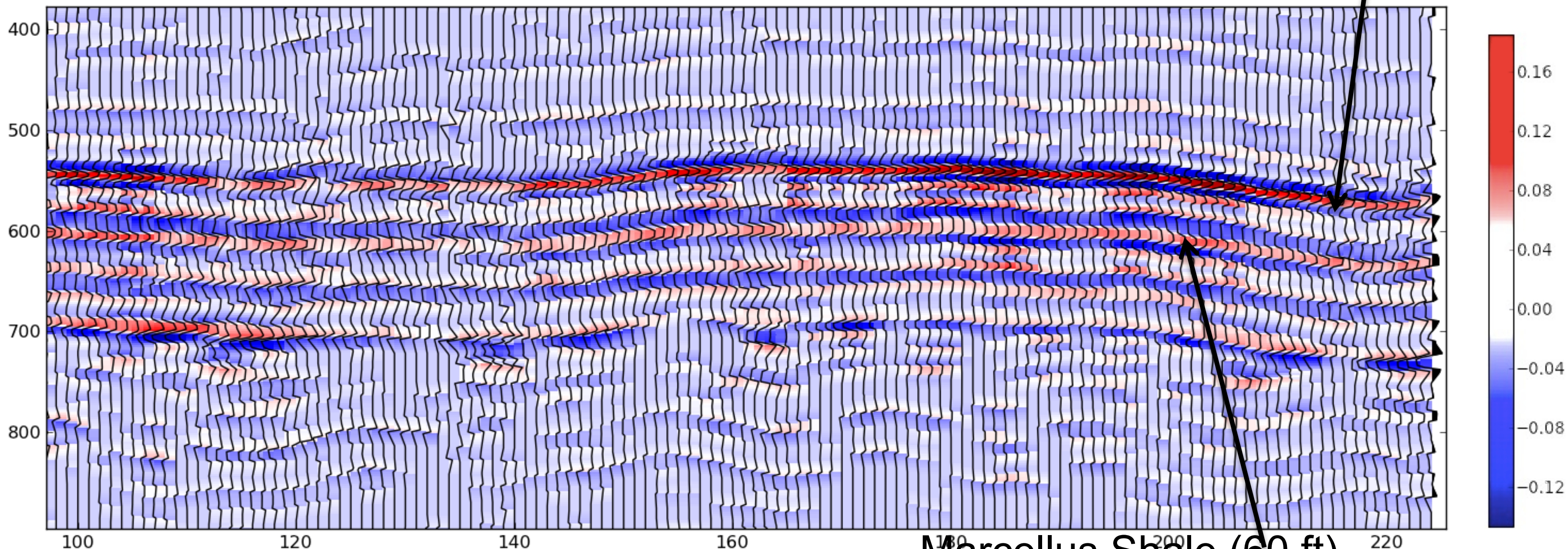
Original Post-Stack Seismic



35 Hz center frequency data

Tully Limestone (50 ft)
(6 ms ~1/5th T)

AI – no background trend



Marcellus Shale (60 ft)
(12 ms)~2/5th T

Summary/Conclusions

- Novel sparse-spike algorithm with proven convergence.
- Scale-space representation of impedance allows for efficient upscaling of logs.
- Quantitative estimates of signal bandwidth above noise dictate model complexity.
- Sparse-spike inversions reconcilable with upscaled well logs.
- Comparable thin-bed resolution to other techniques with a thin-bed prior.

→ Charged to innovate. Driven to solve.™