Stochastic Inversion of Seismic PP and PS Data for Reservoir Parameter Estimation

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Outline

- Background
- Hierarchical Bayesian model for joint inversion
- Synthetic case study based on field data
- Summary and conclusions
Delivery: Open-source Java software for inversion of seismic PP data

- Each layer is modeled as a mixture of permeable (sand or carbonate) and impermeable rock (shale or mudstone).

- The ratio of permeable to impermeable rock is determined by net-to-gross (NG).

- Each permeable rock may include one of four fluid types (oil, gas, brine, or low-saturation gas).

(Gunning and Glinsky, 2004)
Delivery: Data and unknown variables

- **Data**
  - PP traces as functions of incident angles ($S_{pp}$)
  - PP time registration with uncertainty ($T_{pp}$)

- **Unknown variables**
  - PP travel time to each interface ($t_1, t_2, \ldots, t_n$)
  - Permeable rock: Porosity, P-wave and S-wave velocity.
  - Impermeable rock: Density, P-wave and S-wave velocity.
  - Fluid: fluid density and P-wave velocity, fluid saturation.
  - Other unknowns: net-to-gross, layer thickness, etc.
Extension of Delivery for inversion of PP and PS data

- Add two types of seismic data:
  - PS traces in the PS time domain ($S_{ps}$)
  - PS time registration ($T_{ps}$)

- Add two types of unknown variables:
  - PS travel time ($t_{ps}$)
  - PS reflectivity ($R_{ps}$)
Dependent relationships among variables and data (graphical model)

Reservoir or layer parameters ($\alpha$)

- $T_{pp}$
- $T_{ps}$
- $S_{bb}$
- $S_{pp}$
- $d$
- $R_{pp}$
- $v_p$
- $v_s$
- $\rho$
- $t_{pp}$
- $t_{ps}$
- $R_{ps}$
Hierarchical Bayesian model

\[ f(\alpha, t_{pp}, t_{ps}, d, v_p, v_s, \rho, R_{pp}, R_{ps} \mid S_{pp}, S_{ps}, T_{pp}, T_{ps}, D_b) \]

\[ \propto f(S_{pp} \mid t_{pp}, R_{pp}) \quad \text{Likelihood of PP data} \]
\[ \times f(S_{ps} \mid t_{ps}, R_{ps}) \quad \text{Likelihood of PS data} \]
\[ \times f(T_{pp} \mid t_{pp}) \quad \text{Likelihood of PP time registration} \]
\[ \times f(T_{ps} \mid t_{ps}) \quad \text{Likelihood of PS time registration} \]
\[ \times f(D_b \mid d) \quad \text{Likelihood of depth data} \]
\[ \times f(d \mid t_{pp}, v_p) \quad \text{Linkage between PP time and depth} \]
\[ \times f(R_{pp}, R_{ps} \mid v_p, v_s, \rho) \quad \text{Linearized Zoeppritz equations} \]
\[ \times f(t_{ps} \mid t_{pp}, v_p, v_s) \quad \text{Linkage between PS and PP time} \]
\[ \times f(v_p, v_s, \rho \mid \alpha) \quad \text{Rockphysics models} \]
\[ \times f(\alpha)f(t_{pp}). \quad \text{Priors on parameters and PP time} \]
\[ R_{pp}(\theta) = \frac{1}{2} \left( \frac{\Delta v_p}{v_p} + \frac{\Delta \rho}{\rho} \right) \]
\[ + \frac{1}{2} \left[ \frac{\Delta \rho}{v_p} - 4 \left( \frac{v_s}{v_p} \right)^2 \left( \frac{\Delta \rho}{\rho} + 2 \frac{\Delta v_s}{v_s} \right) \right] \theta^2 + O(\theta^4), \]
\[ R_{ps}(\theta) = -\frac{1}{2} \left[ \frac{\Delta \rho}{\rho} + 2 \left( \frac{v_s}{v_p} \right) \left( \frac{\Delta \rho}{\rho} + 2 \frac{\Delta v_s}{v_s} \right) \right] \theta + O(\theta^3). \]

Where \( v_p = (v_{p1} + v_{p2}) / 2, \ v_s = (v_{p1} + v_{p2}) / 2, \ \rho=\left(\rho_1+\rho_2\right)/2, \)
\( \Delta v_p = v_{p2} - v_{p1}, \ \Delta v_s = v_{s2} - v_{s1}, \) and \( \Delta \rho = \rho_2 - \rho_1. \)
Linkage between PP and PS travel time

- Find an interface on which both PP and PS have strong reflection.
- Use the PS time on the interface as the reference to calculate PS time for other interfaces.
- Relative PP and PS time for a given layer is calculated by

\[
\Delta t_{ps} = \frac{1}{2} \left( 1 + \frac{v_p}{v_s} \right) \Delta t_{pp}
\]
Floating-grain rockphysics model by Gunning and Glinsky (2007)

\[ \nu_p = a_{vp} + b_{vp} Z + c_{vp} X + \epsilon_{vp}, \quad \epsilon_{vp} \sim N(0, \sigma_{vp}^2) \]
\[ \nu_s = a_{vs} + b_{vs} \nu_{vp} + \epsilon_{vs}, \quad \epsilon_{vs} \sim N(0, \sigma_{vs}^2) \]
\[ \rho = a_{\rho} + b_{\rho} \nu_{vp} + c_{\rho} X + \epsilon_{\rho}, \quad \epsilon_{\rho} \sim N(0, \sigma_{\rho}^2) \]

where \( Z \) is the loading depth or other variable representing the effect of pressure, and \( X \) is floating grain fraction. All the coefficients and variance are obtained from fitting of borehole logs.
Borehole logs from Gunning and Glinsky (2007)

(a) V_p (km/s) vs. Depth TVD
(b) V_s (km/s) vs. Depth TVD
(c) Density (g/cc) vs. Depth TVD
(d) V_p/V_s vs. Depth TVD
(e) Impedance (MPa) vs. Depth TVD

Upper Pay (Layer-4)
Lower Pay (Layer 6)
<table>
<thead>
<tr>
<th>Geology</th>
<th>Vp (km/s)</th>
<th>Vs (km/s)</th>
<th>Rho (g/cc)</th>
<th>Vp/Vs</th>
<th>NG</th>
<th>Porosity</th>
<th>Floating Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marl</td>
<td>3.67</td>
<td>1.75</td>
<td>2.54</td>
<td>2.10</td>
<td>0.00</td>
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<td></td>
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<tr>
<td>Silt Marl Mix</td>
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<td>1.17</td>
<td>2.38</td>
<td>2.44</td>
<td>0.00</td>
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<tr>
<td>Bounding Shale</td>
<td>3.32</td>
<td>1.63</td>
<td>2.50</td>
<td>2.04</td>
<td>0.00</td>
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<tr>
<td>Sand / Mixing Shale</td>
<td>3.49</td>
<td>1.89</td>
<td>2.39</td>
<td>1.84</td>
<td>0.65</td>
<td>0.187</td>
<td>0.035</td>
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<tr>
<td>Bounding Shale</td>
<td>3.48</td>
<td>1.76</td>
<td>2.52</td>
<td>1.98</td>
<td>0.00</td>
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<td></td>
</tr>
<tr>
<td>Sand / Mixing Shale</td>
<td>3.58</td>
<td>1.97</td>
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<td>1.81</td>
<td>0.65</td>
<td>0.181</td>
<td>0.035</td>
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PP and PS reflectivities and seismic data

<table>
<thead>
<tr>
<th></th>
<th>Full PP Stack ($\theta=0^\circ$)</th>
<th>Full PS Stack ($\theta=45^\circ$)</th>
<th>AVO Gradient Stack ($\theta=45^\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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<td>3</td>
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<td>5</td>
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<td>6</td>
<td>-0.0096</td>
<td>-0.0542</td>
<td>-0.0571</td>
</tr>
</tbody>
</table>

(Sassen & Glinsky, 2013)
Priors about floating-grain fraction

- **Strong prior**: $X \sim N(0.02, 0.03^2)$, True $X=0.035$
- **Weak prior**: $X \sim N(0.0, 0.05^2)$, True $X=0.035$

<table>
<thead>
<tr>
<th>Color</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black</td>
<td>Priors</td>
</tr>
<tr>
<td>Red</td>
<td>Full PP</td>
</tr>
<tr>
<td>Green</td>
<td>Full PP plus AVO gradient</td>
</tr>
<tr>
<td>Blue</td>
<td>Full PP plus full PS</td>
</tr>
</tbody>
</table>

Floating grain fraction

Porosity NG

Strong prior: $X \sim N(0.02, 0.03^2)$, True $X=0.035$

Weak prior: $X \sim N(0.0, 0.05^2)$, True $X=0.035$
Prior about net-to-gross

- **Black**: Priors
- **Red**: Full PP
- **Green**: Full PP plus AVO gradient
- **Blue**: Full PP plus full PS

**Strong prior**: $NG \sim N(0.6, 0.1^2)$, True $NG = 0.65$

**Weak prior**: $NG \sim N(0.5, 0.3^2)$, True $NG = 0.65$
Differences between the true values and estimated medians

Net-to-gross

Porosity

Floating-grain fraction

Vs

Vp

Thickness

Density

Normalized Differences

Prior | Full PP | Full PP & AVO | Full PP & PS | All Data

Normalized Differences

Prior | Full PP | Full PP & AVO | Full PP & PS | All Data

Net-to-gross

Porosity

Floating-grain fraction

Vs

Vp

Thickness

Density

Normalized Differences

Prior | Full PP | Full PP & AVO | Full PP & PS | All Data
Probability of small regions around the true values

\[ \text{Prob}(\theta \in [0.95\theta^{\text{True}}, 1.05\theta^{\text{True}}] \mid \text{Data}) \]
Summary and conclusions

- We developed a tool to combine PP and PS data by extending ‘Delivery’ to include PS responses and time registration as data.

- The revised codes take full advantage of Delivery in model specification, Markov chain Monte Carlo (MCMC) sampling, and post analysis.

- We applied the codes to a synthetic model based on actual borehole logs. We used a floating-grain rockphysics model to link reservoir parameters to seismic attributes.

- The case study results show that full PS data provide more information than AVO gradient data. Specifically, PS data significantly improve the estimates of floating-grain fraction and porosity.
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