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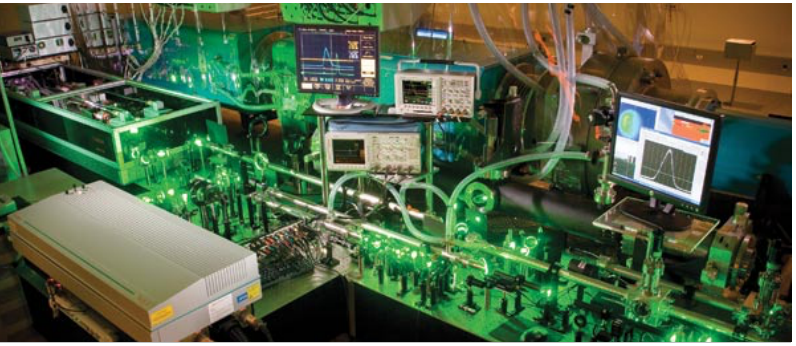

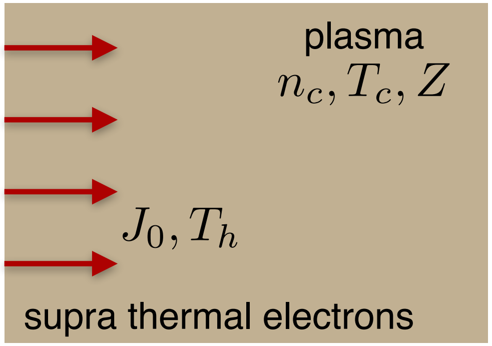

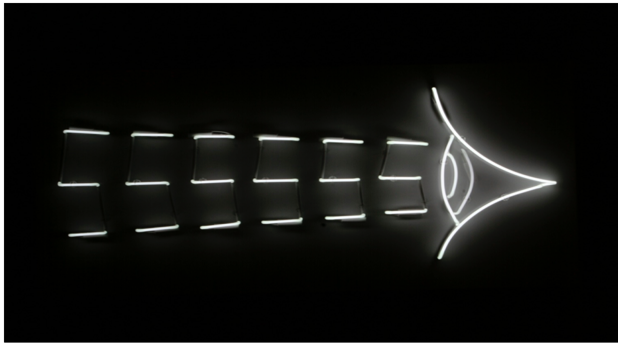

Regimes of suprathermal electron transport*

Michael E. Glinsky

U.S. DEPARTMENT OF ENERGY

NNSA National Nuclear Security Administration

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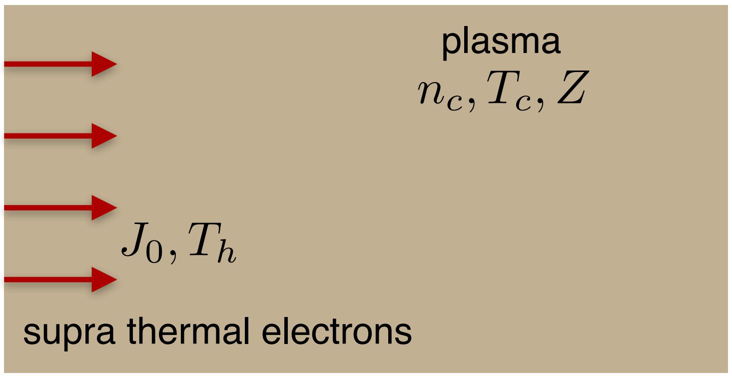
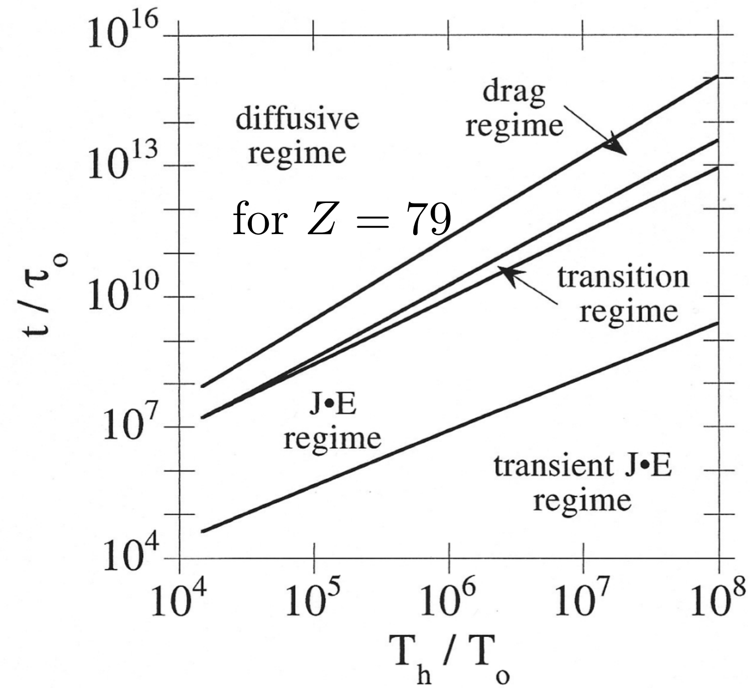
Five simple scaling solutions



$$u_0 \equiv \frac{J_0}{e n_c} \qquad T_0 \equiv m u_0^2$$

$$b_0 \equiv e^2/T_0 \qquad 1/\tau_0 \equiv n_c b_0^2 u_0 Z \ln \Lambda$$



Glinsky, Phys. Plasmas **2**, 2796 (1995).

predicts:

- fast ion generation
- magnetic field generation
- self similar plasma conditions
- E-field transport inhibition

Outline of presentation

- derivation of basic equations
- five regimes of superthermal electron transport:
  - free streaming, transient J.E
  - J.E
  - J.E to cold electron collisional drag transition
  - cold electron collisional drag
  - diffusive
- applications:
  - fast ion generation
  - magnetic field generation
  - self similar plasma conditions
  - E-field transport inhibition

Starting point for physics

Start with collisional Vlasov equation

$$\frac{\partial f_\alpha}{\partial t} + \vec{v} \bullet \frac{\partial f_\alpha}{\partial \vec{r}} + \frac{Z_\alpha e}{m_\alpha} \vec{E} \bullet \frac{\partial f_\alpha}{\partial \vec{v}} = \sum_\beta C\left(f_\alpha, f_\beta\right)$$

using Fokker-Planck collision operator and three species

$$f_i = f_{0i}(v) \qquad \text{(ions)}$$

$$f_c = f_{0e}(v) + \hat{v} \bullet \delta\vec{f}(v) \qquad \text{(cold electrons)}$$

$$f_h = n_h\, g(\Omega)\, \delta\left(v - \sqrt{2T_h / m}\right) \qquad \text{(hot electrons)}$$

Full moment equations

cold electron equations

thermoelectric pressure

hot-cold electron collisional drag

$$\mathbf{E} = \frac{\mathbf{J}}{\sigma} - \frac{5}{2}\frac{1}{e}\nabla T_c - \frac{\mathbf{R}_h}{e n_c}$$

(momentum)

J•E heating

thermal diffusion

hot-cold electron collisional drag heating

$$\frac{3}{2} n_c \frac{\partial T_c}{\partial t} = \nabla \cdot (\kappa \nabla T_c) + \frac{J^2}{\sigma} - \frac{\mathbf{J}\cdot\mathbf{R}_h}{e n_c} + Q_h + \frac{3}{2}\frac{1}{e}\mathbf{J}\cdot\nabla T_c + \frac{5}{2}\frac{T_c}{e}\nabla\cdot\mathbf{J}$$

(energy)

where

$$\sigma \equiv \frac{4\sqrt{2}}{\pi^{3/2}} \frac{T_c^{3/2}}{Z e^2 m_e^{1/2} \ln\Lambda} \qquad \mathbf{R}_h = -\left(\frac{m}{e\tau_{ch}}\right)\mathbf{J}_h$$

$$\tau_{ch} \equiv \frac{3\sqrt{3}}{8\pi} \frac{m_e^{1/2} T_h^{3/2}}{n_c e^4 \ln\Lambda} \qquad Q_h = \tfrac{3}{2} T_h n_h / \tau_{ch}$$

$$\kappa \equiv \frac{16\sqrt{2}}{\pi^{3/2}} \frac{T_c^{5/2}}{Z e^4 m_e^{1/2} \ln\Lambda}.$$

hot electron equations

$$m_e n_h \frac{d\mathbf{u}_h}{dt} + m_e \mathbf{u}_h \left(\frac{\partial n_h}{\partial t} + \nabla\cdot(n_h \mathbf{u}_h)\right) = -\nabla p - \nabla\cdot\mathbf{\Pi} - e n_h \mathbf{E} - \frac{Z}{2}\frac{n_h \mathbf{u}_h}{\tau_{ch}}$$

(momentum)

$$\frac{3}{2} T_h \left(\frac{\partial n_h}{\partial t} + \nabla\cdot(n_h \mathbf{u}_h)\right) = -e n_h \mathbf{E}\cdot\mathbf{u}_h - Q_h$$

(energy)

where

$$p \equiv m_e n_h \langle (\mathbf{v} - \mathbf{u}_h)^2 / 3 \rangle$$

$$\mathbf{\Pi} \equiv m_e n_h \left\langle \mathbf{vv} - \mathsf{I}\,\frac{(\mathbf{v}-\mathbf{u}_h)^2}{3} \right\rangle$$

Reduced set of equations

now assume:

1. $n_c = \text{constant} \gg n_h$
2. $T_h \gg T_c$
3. $J = -J_h$ (quasi-neutrality)
4. $u_h \ll \sqrt{3T_h/m_e}$

which gives the following set of 1D equations:

$$E = \frac{en_h u_h}{\sigma(T_c)}$$ (cold electron momentum, Ohm's law)

$$\frac{3}{2}n_c \frac{\partial T_c}{\partial t} = \frac{\partial}{\partial x}\left[\kappa(T_c)\frac{\partial T_c}{\partial x}\right] + \frac{(en_h u_h)^2}{\sigma(T_c)} + \frac{3}{2}n_h T_h/\tau_{ch}$$ (cold electron energy)

$$0 = T_h \frac{\partial n_h}{\partial x} + en_h E + \frac{Z}{2}n_h m_e u_h/\tau_{ch}$$ (hot electron momentum)

$$\frac{\partial n_h}{\partial t} + \frac{\partial}{\partial x}(n_h u_h) = -\frac{(en_h u_h)E}{3\,T_h/2} - n_h/\tau_{ch},$$ (hot electron energy)

$$u_h \leq \sqrt{3T_h/m_e}$$ (hot electron flux limit)

Dimensionless equations

$$\frac{\partial T_c}{\partial t} = C\frac{\partial}{\partial x}\left(T_c^{5/2}\frac{\partial T_c}{\partial x}\right) + \left(\frac{2A}{3}\right)T_c^{-3/2}\left(n_h u_h\right)^2 + \frac{2B}{Z}n_h T_h^{-1/2},$$

$$\frac{\partial n_h}{\partial t} = -\frac{\partial}{\partial x}\left(n_h u_h\right) - \left(\frac{2A}{3}\right)\frac{T_c^{-3/2}}{T_h}\left(n_h u_h\right)^2 - \frac{2B}{Z}n_h T_h^{-3/2}$$

two coupled 1D nonlinear parabolic partial differential equations

$$n_h u_h^* = -T_h \frac{\partial n_h}{\partial x} \Big/ \left(B T_h^{-3/2} + A n_h T_c^{-3/2}\right)$$

$$u_h = \frac{u_h^* \sqrt{3T_h}}{u_h^* + \sqrt{3T_h}}$$

using these variables to scale the quantities:

$$u_0 \equiv \frac{J_0}{e n_c} \qquad T_0 \equiv m u_0^2$$

$$b_0 \equiv e^2 / T_0 \qquad 1/\tau_0 \equiv n_c b_0^2 u_0 Z \ln \Lambda$$

$$A \equiv \pi^{3/2} / 4\sqrt{2}$$
$$B \equiv 4\pi / 3\sqrt{3}\,.$$
$$C \equiv 32\sqrt{2} / 3\pi^{3/2}$$

and boundary conditions:

$$\frac{\partial T_c(x=0)}{\partial x} = \frac{\partial T_c(x=\ell)}{\partial x} = \frac{\partial n_h(x=\ell)}{\partial x} = 0 \qquad n_h u_h(x=0) = 1$$

Simple scalings

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$$\frac{\partial T_c}{\partial t} \sim \overset{\text{thermal diffusion}}{O\left(\frac{T_c^{7/2}}{L_T^2}\right)} + \overset{\text{J•E heating}}{O\left(T_c^{-3/2}\right)} + \overset{\text{hot-cold electron drag heating}}{O\left[\frac{2B}{Z} n_h T_h^{-1/2}\right]},$$

$$\frac{\partial n_h}{\partial t} \sim \overset{\text{advection}}{O\left(\frac{1}{L_n}\right)} - \overset{\text{J•E heating}}{O\left(\frac{T_c^{-3/2}}{T_h}\right)} - \overset{\text{drag heating}}{O\left[\frac{2B}{Z} n_h T_h^{-3/2}\right]},$$

$$L_n \sim \min\left[\overset{\text{hot electron resistivity}}{\frac{n_h T_h^{5/2}}{B}}, \overset{\text{cold electron resistivity}}{T_h T_c^{3/2}}, \overset{\text{flux limit}}{n_h \left(3T_h\right)^{1/2} L_n}\right].$$

where the density and thermal scale lengths are defined as:

$$L_n \sim n_h / \left(\partial n_h / \partial x\right)$$

$$L_T \sim T_c / \left(\partial T_c / \partial x\right)$$

Regimes determined by which terms dominate

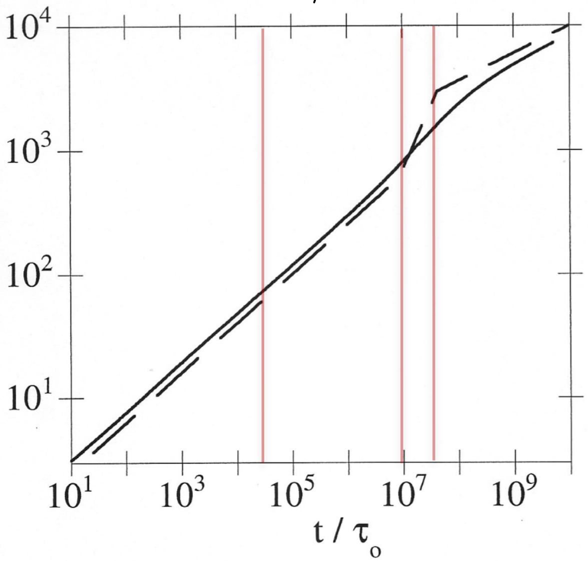
	transient J•E	J•E	drag–J•E transition	drag	diffusion
$T_c \sim$	$t^{2/5}$	$t^{2/5}$	$t^{2/5}$	$T_h^{-1}\left(\frac{Z}{2B^2}\right)^{-1/2} t$	$T_h^{4/9}\, t^{2/9}$
$L_n \sim$	$(3T_h)^{1/2} t$	$T_h^1\, t^{3/5}$	$T_h^1\, t^{3/5}$	$T_h^2\left(\frac{Z}{2B^2}\right)^{1/2}$	$T_h^2\left(\frac{Z}{2B^2}\right)^{1/2}$
$L_T \sim$	$(3T_h)^{1/2} t$	$T_h^1\, t^{3/5}$	$T_h^1\, t^{3/5}$	$T_h^2\left(\frac{Z}{2B^2}\right)^{1/2}$	$T_h^{5/9}\, t^{7/9}$
$n_h \sim$	$(3T_h)^{-1/2}$	$T_h^{-1}\, t^{2/5}$	$\left(\frac{Z}{2B}\right) T_h^{1/2}\, t^{-3/5}$	$\left(\frac{Z}{2}\right)^{1/2} T_h^{-1/2}$	$\left(\frac{Z}{2}\right)^{1/2} T_h^{-1/2}$
$t_{\min} \sim$	0	$(T_h/3)^{5/4}$	$T_h^{3/2}\left(\frac{Z}{2B}\right)$	$T_h^{5/3}\left(\frac{Z}{2B^2}\right)^{5/6}$	$T_h^{13/7}\left(\frac{Z}{2B^2}\right)^{9/14}$

$\frac{\partial T_c}{\partial t} \sim$	J•E	J•E	drag	drag	diffusion
$\frac{\partial n_h}{\partial t} \sim$	advection	advection	advection vs. drag	advection vs. drag	advection vs. drag
$L_n \sim$	flux limit	cold electron resistivity	cold electron resistivity	hot electron resistivity	hot electron resistivity

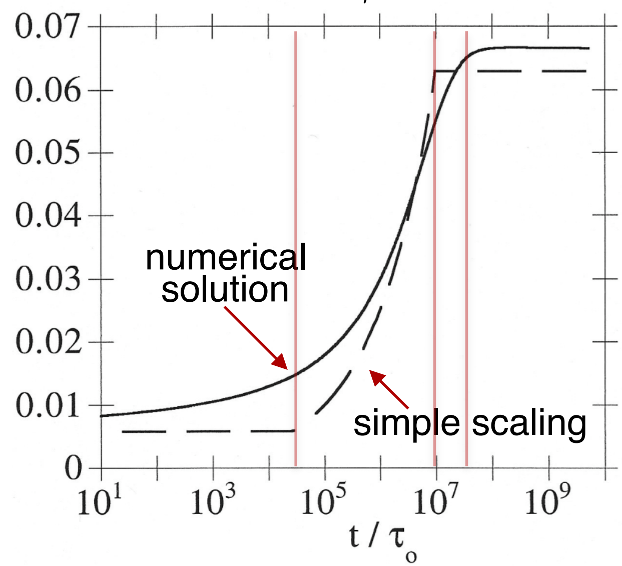
Example solutions

for $Z = 79$ and $T_h/T_0 = 10^4$

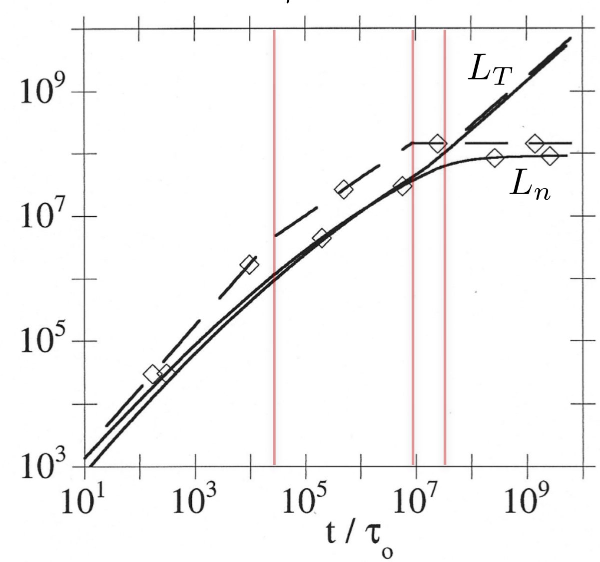
T_c/T_0



n_h/n_c

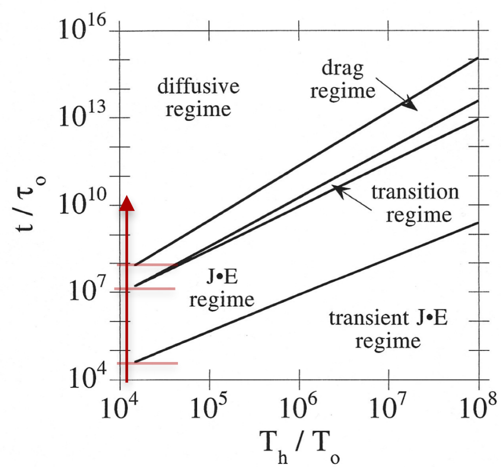


$L/u_0\tau_0$



$L_n \le \sqrt{Z/2B^2}\, T_h^2 \ll \sqrt{3T_h}\, t$

free streaming limit

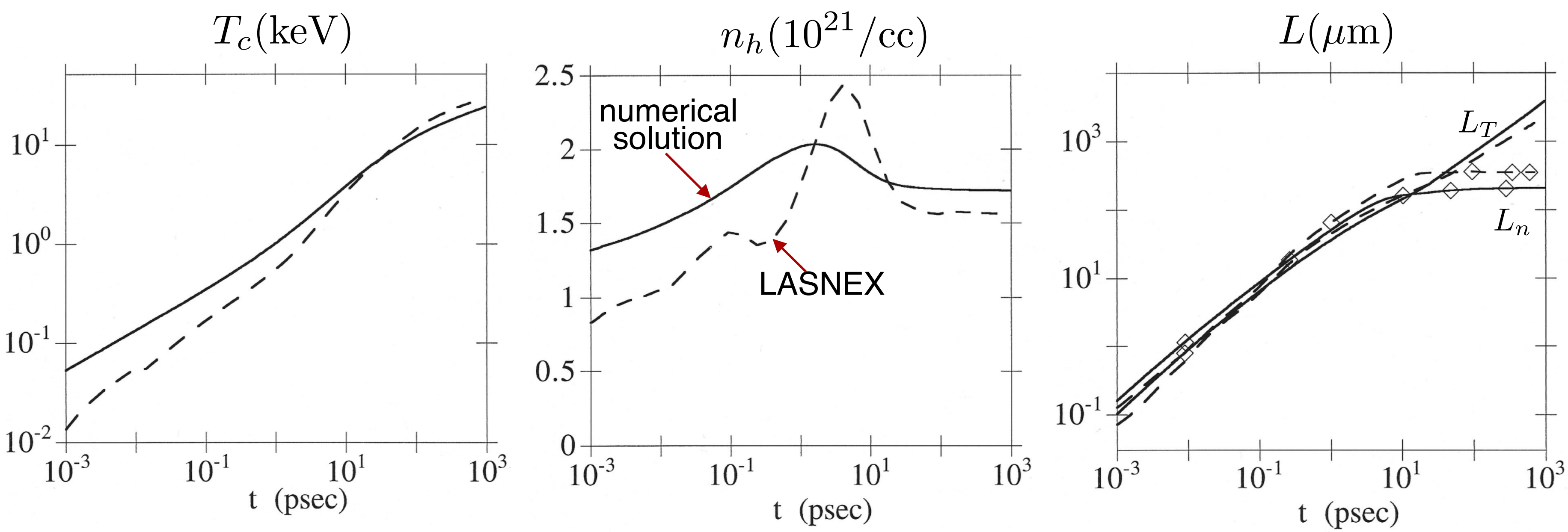


$$(n_h)_{\max} = (Z/2)^{1/2} T_h^{-1/2} \left(\frac{T_h}{B^4 Z/2} \right)^{1/10} \gg (Z/2)^{1/2} T_h^{-1/2}$$

diffusive limit

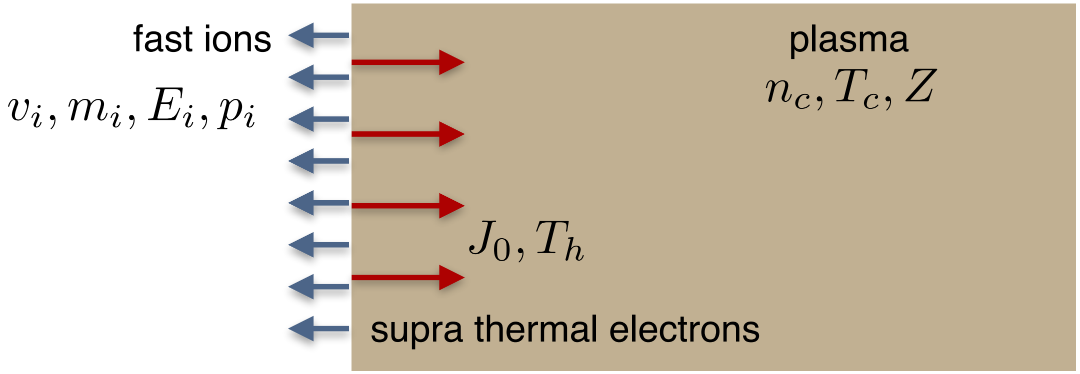
Comparison to LASNEX

$$3 \times 10^{17}\mathrm{W/cm^2}, T_h = 100\mathrm{keV}, n_c = 0.24\mathrm{gm/cc}$$



1D LASNEX with multigroup hot electron diffusion

APP: fast ion generation



momentum and energy exchange at surface:

$$\frac{1}{\text{area}}\frac{dp_i}{dt} = (n_h v_h)(m_e v_h)$$

$$\frac{1}{\text{area}}\frac{dE_i}{dt} = v_i \frac{1}{\text{area}}\frac{dp_i}{dt} \sim n_h T_h v_i$$

ion velocity from two expressions for ion energy:

$$\frac{n_h}{Z}(m_i v_i^2)\, d \sim \int_0^t \frac{1}{\text{area}}\frac{dE_i}{dt}\, dt \sim n_h T_h d$$

$$v_i \sim \left(\frac{Zm_e}{m_i}\right)^{1/2}\left(\frac{T_h}{m_e}\right)^{1/2}$$

compare to rate of hot electron production:

$$\frac{dE_i / dt}{dE_h / dt} \sim \left(\frac{n_h}{n_c}\right)\left(\frac{T_h}{T_o}\right)^{1/2}\left(\frac{Zm_e}{m_i}\right)^{1/2}$$

use of n_h scaling gives:

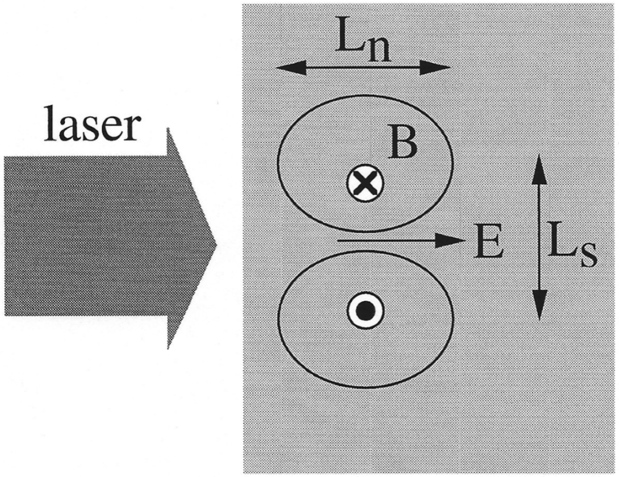
$$\boxed{\left(\frac{dE_i / dt}{dE_h / dt}\right)_{\max} \sim \left(\frac{Z}{2}\right)^{1/2}\left(\frac{Zm_e}{m_i}\right)^{1/2}\left[\left(\frac{T_h}{T_o}\right)\frac{1}{B^4 Z/2}\right]^{1/10}}$$

$$1\,\text{ps}, 10^{17}\,\text{W/cm}^2, 80\,\text{keV}, Z^* = 25 \text{ for Au}$$

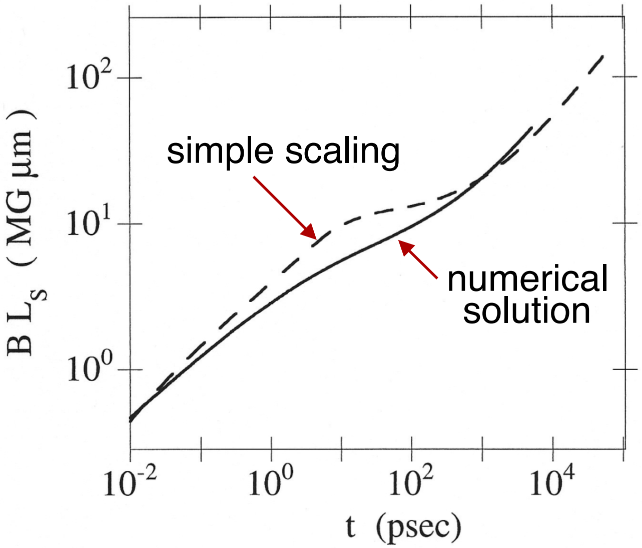
	simple model	LASNEX
hydrogen	4 %	4 %
solid gold	12 %	11 %
hydrogen on solid gold	18 %	18 %

500Å

APP: magnetic field generation



$3 \times 10^{17} \mathrm{W/cm^2}, T_h = 100 \text{ keV}, n_c = 0.24 \mathrm{gm/cc}$



from Maxwell's equations:

$$\dot{B} = -c\nabla \times E \sim cE/L_s$$

from Ohm's law for cold electrons:

$$E = B_0(AZ \ln \Lambda)(T_c/T_0)^{-3/2}$$

$$B_0 \equiv e^3 n_c / T_0$$

APP: similar plasma conditions

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● $I = 10^{19}\mathrm{W/cm}^2, \lambda = 0.1\mu\mathrm{m}, \tau_{\mathrm{pulse}} = 1\mathrm{ps}, n_c = 10^{26}/\mathrm{cc}$

● $I_s = 10^{15}\mathrm{W/cm}^2, \lambda_s = 1\mu\mathrm{m}, \tau_s = 100\mathrm{ps}, n_c = 10^{23}/\mathrm{cc}$

from Kruer & Eastbrook:

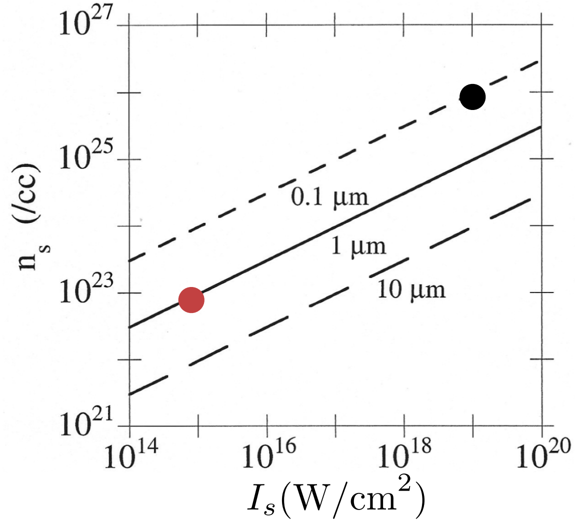
$$T_h \sim (I\lambda^2)^{1/3}$$

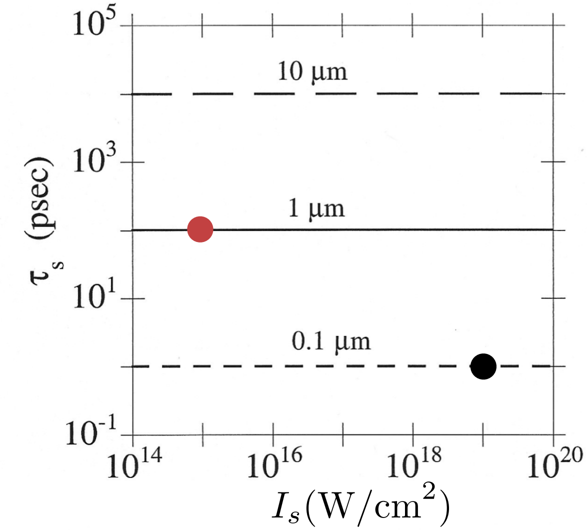
given base conditions, find:

$$I_s, \lambda_s \rightarrow n_s, \tau_s$$

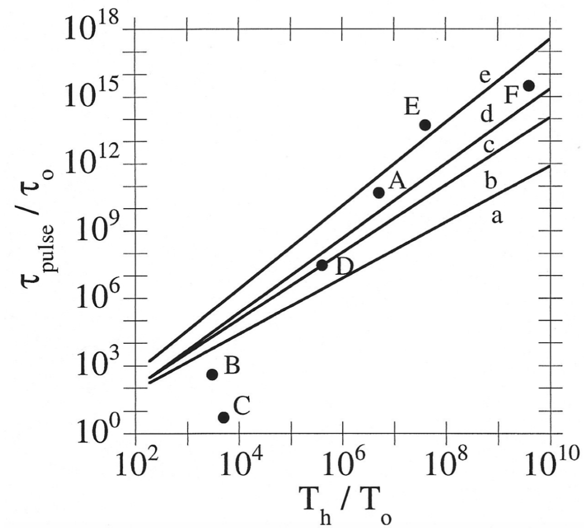
$$n_s = n_c \left(\frac{I_s}{I}\right)^{1/2} \left(\frac{\lambda_s}{\lambda}\right)$$

$$\tau_s = \tau_{\mathrm{pulse}} \left(\frac{\lambda_s}{\lambda}\right)^{-1}$$



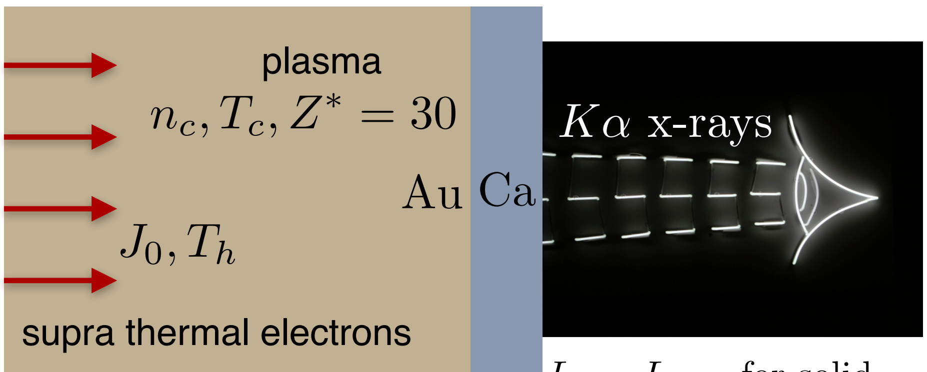


condition	I (W/cm^2)	t_{pulse} (psec)	n_c (/cc)	T_h (keV)
A	10^{16}	750	10^{22}	200
B	10^{18}	1	10^{23}	200
C	10^{19}	0.1	10^{23}	1000
D	10^{19}	1	10^{26}	100
E	10^{15}	100	10^{24}	10
F	10^{16}	750	10^{24}	200

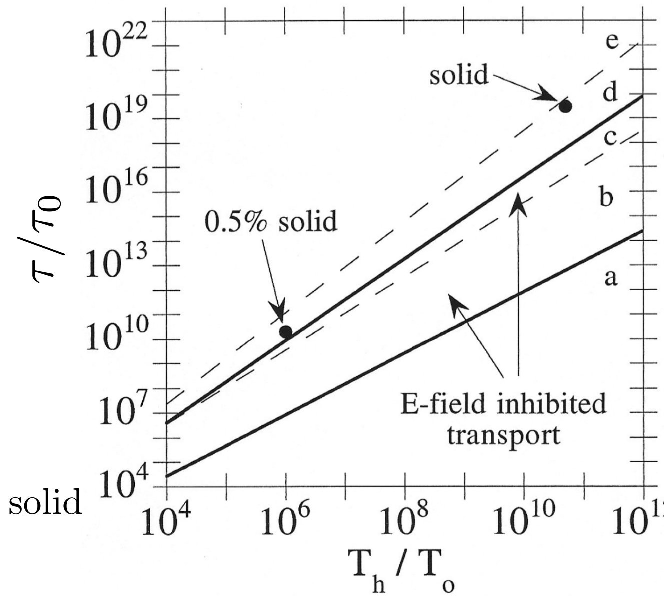


APP: E-field transport inhibition

2% absorption of $3 \times 10^{15} \mathrm{W/cm}^2, \tau = 100$ ps, $\lambda = 1\mu$m, $T_h = 14$ keV



$L_n = L_{max}$ for solid
$L_n = 0.25 L_{max}$ for 0.5 % solid



gold foil targets of solid and 0.5% solid density

$$L_n \ll \min[(3T_h)^{1/2} t, (Z/2B^2)^{1/2} T_h^2] = L_{max} \text{ if E-field inhibited transport}$$

Bond, Hares, Kilkenny, PRL **45**, 252 (1980).

Beg et al., Phys. Plasmas **4**, 447 (1997). (for 10^{19} $\mathrm{W/cm}^2$)

Acknowledgments

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