Wavelet Extraction: an essay in model selection and uncertainty

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Outline

- Model selection, inference and prediction: Generalities
- Wavelet extraction
 - Model formulation
 - Generic inversion/inference features for single models
 - Model choice scenarios. 3 examples
- Morals & conclusions

Model selection

- Classic statistics problem
 - line ? or parabola? or sextic?
- . Foundations of science
 - Newton or Ptolemy?





F = m **a**



Generalities

- . Geostatistics is only one branch of multivariate statistics.
 - Every result of significance can be derived from conventional multivariate estimation theory. A Bayesian spin is helpful.
 - The rest of the world understands us better if we write things this way.
- . As statisticians, we have the duties of
 - Model inference
 - Significance testing
 - Model refinement/testing
 - Making predictions with error estimates

Model selection & Bayes factors

- Suppose there are $k{=}1{\ldots}N$ models to explain data D, with parameters m_k
- The model has prior $P(m_k|k)P(k)$, likelihood $L(D|m_k,k)$
- Here's Three very useful things:
 - Marginal model likelihood:

 $P(D|k) = \int L(D|m_k, k) P(m_k|k) dm_k$

- Bayes factor $B_{ij} = P(D|i)/P(D|j)$
- Model probability of model k:

$$P(k \mid D) = \frac{P(D \mid k)P(k)}{\sum_{j=1}^{N} P(D \mid j)P(j)}$$

Predictive distributions

- Bayesian model averaging: prediction for new y •
 - $P(y|D) = \sum_{k} P(y|M_{k},D) P(D|k)$



- Mixture distributions
 - Models/rock-types/properties
 - Most helpful way to manage heavy tails
- Often: •
 - uncertainties *within* a model < uncertainty between models

Noise estimation

- Noise $\varepsilon \equiv (D-f(m))$
 - $\varepsilon = \varepsilon_{meas} + \varepsilon_{model}$
 - Often $\varepsilon_{model} > \varepsilon_{meas}$: we're prepared to live with approximate models
 - ϵ_{meas} usually stationary. Maybe biased. Roughly known.
 - ϵ_{model} often nonstationary. Correlated. Often biased. Usually unknown.
 - We blaze away: $\{\epsilon, \sigma^2\} \sim N(0, \sigma^2)P(\sigma^2)$
- Inference of $p(\epsilon)$ chief issue
 - p(ϵ) strongly coupled to model dimensionality
 - Demands sensible priors in Bayesian formulation
 - Simplest form adds pieces like
 - $Log(\Pi) \sim |D-f(m)|^2/(2\sigma^{2)}+n_D \log(\sigma^2)/2$
 - Absolute statements of model significance no longer possible

Marginal model likelihood estimators

- Full linearisation+conjugate priors, analytical
- Quadratures. Too hard as d=*dim*(m) gets large
- MCMC "Harmonic mean" averages:

$$P(D | k) = \int L(D | m_k, k) P(m_k, k) dm_k \qquad (1)$$

$$\hat{P}(D | k) = \left\langle \frac{1}{L(D | m_k, k)} \right\rangle_{\Pi(m_k | D)(MCMC)}^{-1}$$
Unstable as Nsamples $\rightarrow \infty$

· Laplace-Metropolis approx.

 $P(D \mid k) \approx \int e^{-f(m)} dm \approx (2\pi)^{d/2} \det\{\frac{\partial^2 f}{\partial m_i \partial m_j}\Big|_{\hat{m}}\}^{-1} e^{-f(\hat{m})} = (2\pi)^{d/2} |H|^{1/2} L(D \mid \hat{m}) P(\hat{m})$

• BIC asymptotics (eqn (1) does the "overfit" penalty automatically) $-\log(P(D \mid k)) \sim -\log(L(D \mid \hat{m})P(\hat{m}))) + \frac{d}{2}\log(n_D) + O(n^{-1/2})$

Issues in prior formulation

- Lindley paradox (1957)
 - Models M_1 and M_2 , with $d_2 > d_1$. Suppose they share (nested) parameters m with prior

 $m_1 \sim N(0, \sigma^2 C_1)$

 $m_2 \sim N(\{0,0\},\sigma^2 diag\{C_1,C_2\})$

 $P(\sigma) \sim 1/\sigma$ (Jeffrey's prior)

Then the Bayes factor is (Smith & Spiegelhalter 1980)

$$\begin{split} B_{12} &\sim |C_2|^{1/2} |X_2^T X_2|^{1/2} (1 + ((d_2 - d_1)/(n - d_1)F)^{n/2} \\ \text{Always favours simplest model (1) as } |C_2| \rightarrow \infty \\ \text{(even for proper prior on } m_2!) \end{split}$$

• Setting of reasonable prior variances *very* important in comparing different dimensional models

Segmentation/Blocking algorithms

- Changepoints a classic modelchoice problem
- Maximum likelihood configurations for k changepoints τ_j accessible from dynamic programming O(kn²) (D.M.Hawkins, Comp. Stat. & Data Analysis 37(3), 2001)
- Uses:
 - Thinning reflectivity sequence
 - Optimal checkshot choices



$$-\log(L(y,m,\tau)) = \frac{1}{2} \sum_{j=1}^{k} \sum_{i=\tau_{j-1}+1}^{\tau_j} (y_i - m_j)^2 / \sigma^2 + \frac{n}{2} \log(\sigma^2) + BIC$$

Wavelet extraction problem



Real wavelets and effective wavelets: Ricker 1953 (dynamite)



FIG. 5. The primary seismic disturbance, velocity type, as observed in Pierre shale.

Motivations

- Wavelet extraction a critical process in inversion studies
- . Commercial codes don't (or wont) tell you
 - Noise estimates
 - Non-normal-incidence wavelets
 - Time to depth or positioning errors
 - May struggle with
 - Multiple wells
 - Deviated wells
 - wavelet uncertainties

Wishlist

- Wavelet coefficient extraction (plus errors)
- Wavelet length estimation (plus errors)
- Noise estimation (plus errors)
- Petrophysics model choices
- Time to depth adjustments(plus errors)
 - Integrates checkshots and sonic logs
- Positioning adjustments (plus errors)
- Multi-well
- Multi-stack

Wavelet Parameterization (1)

- Wavelet coefficients m_w
 - Nyquist-rate knots on cubic splines with zero endpoints & derivatives
 - Ensures decent tapering & correct bandwidth



Checkshot Parameterization (2)

Time to depth parameters



Petrophysics parameterizations: Multi stack



Petrophysics Parameterizations(4): AVO effects (Linearized Zoeppritz)

General approximate p-p reflection coefficient



- Stack angle $\theta^2 = v_p^2 / (V_{stack}^4 T_{stack}^2 / \langle X_{stack}^2 \rangle)$
- R computed at block boundaries: v_p , v_s , ρ are Backusupscaled segment properties (i.e. error-free log data)
- Anisotropy: $\delta \sim N(\delta_f, \sigma_f^2)$, f=lumped facies parameter label (from classification of segment)

Anisotropy issues

- Processing issues for AVO
 - Many potential amplitude effects at wider offset
 - Absorption/scattering/spreading/acquisition footprints, etc.. etc.. etc
 - Ch. 4.3 "Quantitative seismic interpretation" Avseth, Mukerji, Mavko.

Preprocessing: Impedance Blocking

- Based on segmentation of pwave impedance (ρv_p)
 - Max Lik. methods O(N²), or
 - Blended hierarchical stepwise segment/aggregate method (O(Nlog(k))
 - ref: D.M.Hawkins, Comp. Stat. & Data Analysis 37(3), 2001
- Increases speed



Optimisation of Bayesian Posterior

sonic logs"

 m={m_{wavelet}, m_{checkshot}, m_{registration}, m_{petrophysics}} • Maximise $\prod \propto \exp(-\chi^2/2)$

$$\begin{split} \chi^2 &= \sum_{t,stacks,wells} (w(m) * R_{eff} - S)^2 / \sigma_s^2 \quad "good synthetic" \\ &+ (d+1) \sum_{stacks} \log(\sigma_s) \quad "noise normalisation" \\ &+ \sum_{intervals,wells} (v_{int}(m) - \langle v_{int} \rangle)^2 / \sigma_v^2 \quad "prior: interval velocities c.f. sonic logs \\ &+ (m - \langle m \rangle)^T C_p^{-1} (m - \langle m \rangle) \quad "prior on checkshots, wavelet coeffs" \\ &+ wavelet timing/phase prior \quad "e.g. constant phase" \end{split}$$

Optimisation, Hessians, Approximate uncertainties



•Gauss-Newton, BFGS methods: see e.g. Nocedal & Wright "Numerical Optimization" •O(d²) x O(forward model cost) x N(models)

•Typical dimensionalities: d=10-100, n = 100-1000

Optimisation outputs

- Maximum aposteriori parameters
 - SU wavelets, ascii time-to-depth files, visualization dumps etc.
- Covariance diagonals (parameter uncertainties)
- Posterior model probabilities (wavelet span, checkshot choice etc)
- stochastic wavelets from the posterior

Generic aspects of well-ties



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Joint straight-hole & sidetrack

Straight hole detail with logs

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Interval sonic velocities constrained to within 5% of log average

Parameter uncertainty cross sections



General effect of allowing more freedom

• Wavelets sharpen, noise reduces



Cross well issues: Wavelets and Ties

Synthetics/traces from joint extraction



Relations to Cross-validation

Leave-one-out pseudo Bayes factor

$$B_{12(cross-val)} = \prod_{i=1}^{n_D} \frac{p(d_i | d_{\{j \neq i\}}, M_1)}{p(d_i | d_{\{j \neq i\}}, M_2)}$$

- Has asymptotics of Akaike information criterion
 - Log(P(D|k)) += d (c.f. Bayes info. crit. = $(d/2)\log(n_D)$
 - (less severe on richer models)

3 Model Selection examples

(1) Simple Wavelet-length choice



Wavelet model

Absolute significance tests

- MAP Wavelet length *not* shortest model
- MAP noise level exceptional on null-hypothesis test
 - Perform ensemble of extractions on fake seismic data with matched bandwidth and univariate statistics:



(2) Checkshot choice (25 models)



Richest model

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Model probabilities and noise



(3) Naïve facies-driven anisotropy choice

δ~0.15



Conclusions

- Model selection is desirable in nearly all inverse problems
- Marginal model likelihoods can be tricky
 - Careful construction of prior
 - Efficient (differentiable) forward models
 - Efficient optimisers
- Bayesian model selection is quite "opinionated". Very Occamist: severe on overparameterised models
- Selection is a doable problem for wavelet extraction. Produces
 - Often compact wavelets
 - Simplified checkshots
 - Significant potential for missing-data (petrophysics) problems

O me, the word 'choose!' I may neither choose whom I would nor refuse whom I dislike.

Portia, The Merchant of Venice.

More info:

- www.google.com/search?q=James+Gunning+CSIRO
 - "Wavelet extractor" open source code (java), demos, papers.
 - Part of "Delivery" suite (Bayesian seismic inversion)
- Computers and Geosciences **32** (2006)

Data-positioning uncertainty (1)

- y(m) = f(m) an unusual regression problem
- Simple toy problem; fitting a line y=a+bx:
 - y ~ X.m + e, Jeffrey's prior on e ~ $N(0,\sigma)$
 - Priors:
 - P(σ) ~ 1/σ
 - $P(m|\sigma) \sim N(0, g \sigma^2(X^TX)^{-1})$ ("Zellner")
 - Posterior marginal
 - $\prod = \int L(y|m,\sigma)P(m|\sigma)P(\sigma)dmd\sigma \sim (1+g(1-R^2))^{-(n-1)/2}$
 - Where R² is usual coeff. of determination
- . For two samples of "random" y
 - $B_{12} \sim ((1+g(1-R_1^2))/(1+g(1-R_2^2)))^{-(n-1)/2}$

Data-positioning uncertainty (2)

- Fluctuation in B_{12} for problem with
- y=1+x+(25% fixed noise) + (5% fluctuations)



