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Wavelet Extraction – An Essay in Model Selection and Uncertainty

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SUMMARY

Wavelet extraction is a fundamental step in linking imaged seismic data and well-logs. This step provides time-to-depth estimates that help locate seismic data more precisely in depth, and the wavelet that connects the low-resolution seismic data to the fine-scale properties typical of geocellular models. Noise estimates from well ties are the most important parameter controlling the extent to which seismic data constrain these fine-scale models.

In usual practice, many subjective judgements are made regarding issues like wavelet phase, span, rock--physics models, imaging quality etc, all of which make the results less objective than desirable. It is also widely under-appreciated that these subjective decisions have strong impacts on the most important outputs of the extraction process. They propagate far downstream into reservoir prediction or forecasting, and their influence can easily dominate development decisions.

We show that model selection choices relating to wavelet span, rock-physics models, segmentation etc, can be made more objectively using Bayesian model-selection criteria. We present some case studies showing how strongly parameter estimates and uncertainties are coupled to model choice, and thus why a more objective model-selection process is crucial to the wavelet extraction workflow.

1 Introduction

The reservoir characterization field has seen a huge growth in “stochastic–inversion” or “Bayesian seismic inversion” approaches to coupling seismic data to reservoir models in recent years, e.g. Buland et al. (2003); Eidsvik et al. (2004); Gunning and Glinsky (2004). Areas of intense activity include rock–physics models and fluid uncertainty. The critical “modelling glue” in all of these approaches is the estimation of the (effective) source signature of the seismic data and the estimation of the noise parameters describing the difference between modelled and observed seismic data. The most reliable inversions always require local calibration in the form of wavelet extraction performed on wells from the same processed survey, preferably obtained in similar depth ranges and through the same basic geology as the inversion prospect. Appropriately processed seismic data is crucial. Predictive distributions from these inversions are the direct basis for decisions about drilling or subsequent development.

Such inversions can only be as good as the wavelet extraction obtained in the calibration stage. And since the computational effort involved in wavelet extraction is usually orders of magnitude less than that in Bayesian seismic inversion, we believe more effort should be expended in removing sources of subjectivity or bias in wavelet and noise estimation.

Our approach to wavelet extraction (Gunning and Glinsky, 2006) is overtly Bayesian. This yields “best” parameter estimates, model uncertainties, and optimum model choices in a natural way. The Bayesian approach integrates prior knowledge about the well tie in the form of marker constraints, VSP data, phase & timing prejudices, or plausible interval velocities. We usually seek simultaneous extractions at multiple (possibly deviated) wells, for multiple offsets (using a linearised Zoeppritz reflectivity), and estimate additional uncertainty parameters such as time–registration errors for stacks or well–location errors caused by imaging problems. We want also distribution details for the noise amplitude, and multiple realisations of the extracted wavelets from the Bayesian posterior, showing the uncertainty in the wavelet scaling and extent, the time–to–depth map, and the noise parameters for each stack.

2 Theory

The wavelet+welltie model $\mathbf{m}$ we seek comprises wavelet coefficients, noise parameters, and usually some of (i) local and systematic adjustments to the time-to-depth map, (ii) rock physics parameters, and (ii) well–positioning errors. The data D will comprise (i) imaged seismic amplitudes, (ii) checkshot points (iii) log measurements.

Given any particular model $\mathbf{m}$ for the wavelet extraction parameters, the posterior distribution for the model is the usual Bayes relation

$$\Pi(\mathbf{m}|D) \sim L(D|\mathbf{m})p(\mathbf{m})$$

where $L(D|\mathbf{m})$ is the likelihood of the data D given the model $\mathbf{m}$, and $p(\mathbf{m})$ is a sensible prior for the model. The prior $p(\mathbf{m})$ will in general be as non-informative as possible, but specific prejudices in the form of phase or timing constraints may be embedded in it. We assume Gaussian distributions for the noise process associated with seismic amplitudes, and a convolutional forward model, so the log–likelihood $-\log(L(D|\mathbf{m}))$ contains a term

$$-\log(L_{\text{mistie}}) \sim \frac{1}{2} \sum_{\text{wells i, stacks k}} \left\{ \frac{\|(\mathbf{S}_{ik} - \mathbf{R}_{ik}(\mathbf{m}) * \mathbf{w}(\mathbf{m})\|^2}{\sigma_k^2} + (n+1)\log(\sigma_k) \right\}$$

where S is the seismic data, w the wavelet, σ_k is the kth stack noise level, and n is the number of "effective samples" in the well–tie interval. This piece makes synthetic seismics "look like" the actual data. The reflectivity $\mathbf{R}$ is computed from automatically–segmented density and sonic logs using Backus averaging and the linearised Zoeppritz p–p reflectivity appropriate for the stack angle θ of each stack. At a segment boundary,

$$R = \frac{1}{2}\left(\frac{\Delta\rho}{\rho} + \frac{\Delta v_p}{v_p}\right) + \theta^2 \left(\frac{\Delta v_p}{2\,v_p} - \frac{2\,{v_s}^2\left(\frac{\Delta\rho}{\rho} + \frac{2\,\Delta v_s}{v_s}\right)}{{v_p}^2} + \delta^2/2\right), \tag{1}$$

where ρ is density and $v_{p,s}$ are p,s velocities of the blocked well data; δ is a Thomsen anisotropy parameter that may or may not enter the model, depending on the rock physics. There are other likelihood terms in $L(D|\mathbf{m})$ relating to interval velocity constraints.

In the general case we contemplate $k = 1 \dots N_m$ possible models $\mathbf{m}_k$ that might explain the data (various wavelet lengths, rock–physics models, lateral well–positioning freedom etc). Agnostically, we assume all models to have equal prior weight. The posterior space is then the joint space of models and continuous parameters, and each model, in general, is of different dimensions. Among interesting entities to compute is the most likely wavelet model, obtained by ranking the *model marginal likelihoods* $P_k(D) \sim \int \Pi(D|\mathbf{m}_k) d\mathbf{m}_k$.

For linear models, it is well known that the overall model probability computed from the $P_k(D)$ (and the associated Bayes factors formed by quotients of these probabilities when comparing models) has a strong tendency to penalize models that fit only marginally better than simpler models. This measure is a modern version of Occam's razor, and is closely related to the well–known Bayesian information criterion (BIC). The posterior distributions and model marginal likelihoods are computed exhaustively in our approach. Realisations illustrating the uncertainty can be drawn from the resulting "mixture" distribution.

Within this problem, model-selection also appears as a preprocessing problem in optimally blocking the well logs for acoustic impedance. For this problem, the combinatorially explosive number of candidate models can be efficiently computed using dynamic programming algorithms, which are only $O(n^2)$ in the number of log data.

3 Examples

Figure 1 shows a simple example with model selection used to select the wavelet length for a simple, single well, single stack extraction problem. The right insets show the MAP wavelets for each model, and the MAP model noise estimate and model marginal likelihood graphed against the model label. The left inset is the fit at the MAP point for the most-likely model (number 3).

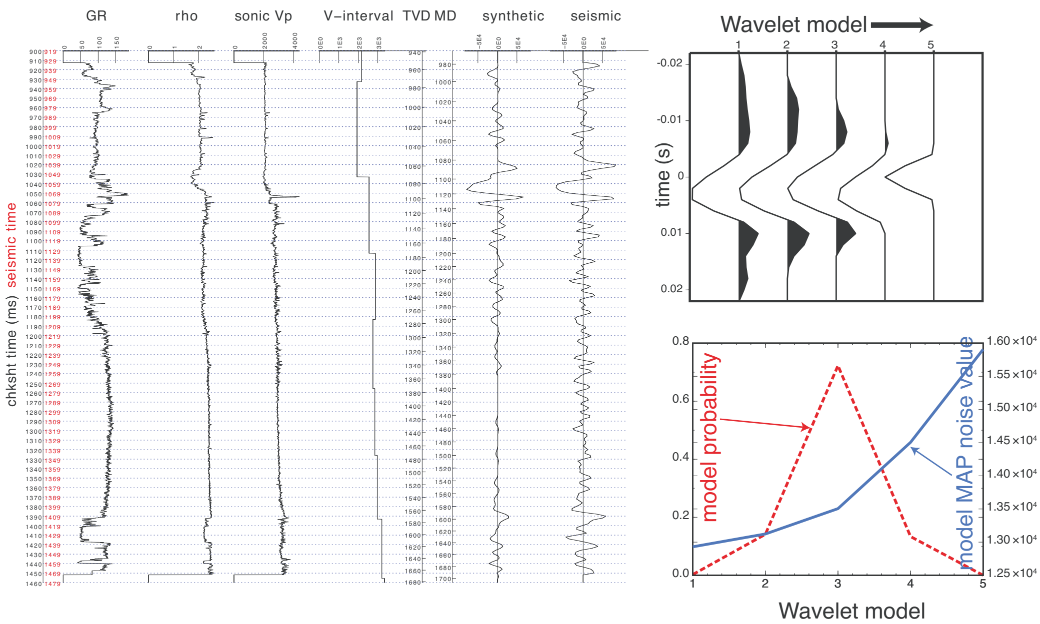

Figure 1: Typical single well extraction with multiple-wavelet models and wavelet probabilities.

Figure 2 shows another simple example where the possibility of anisotropic shales is entertained: this might be helpful in explaining the large far–stack amplitudes observed in a two stack extraction. In this case, the model–selection criterion settled on an anisotropic model with more than 95% probability.

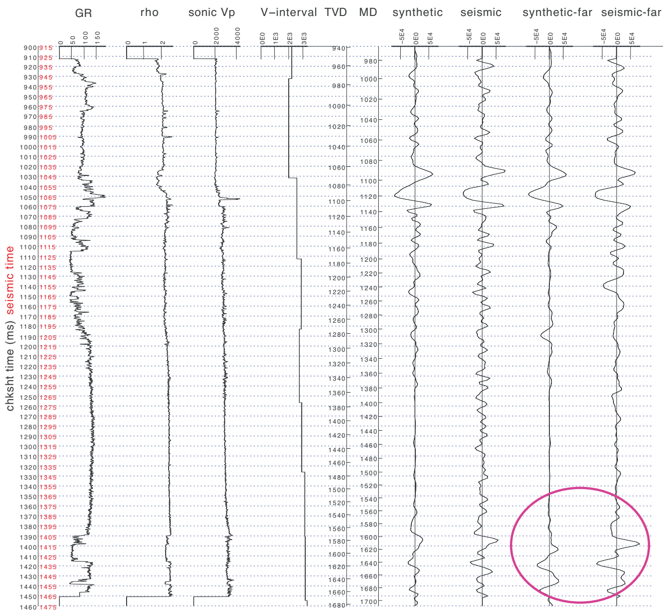

Figure 2: Dual–stack single well extraction testing anisotropy models: far-stack amplitudes in red–circled region can only be produced if an anisotropic rock physics model for shales is chosen.

4 Conclusions

Computing well-ties is a crucial step in parameter estimation of wavelets, noise, time-to-depth relations, and rock physics models, for use in subsequent inversion and prediction calculations. Model uncertainty is a major issue in the well tie procedure. Formal model-selection procedures such as Bayesian approaches are of great help in removing subjective judgements, and nicely encapsulate commonsense notions of statistical significance and the balance between over and under–fitting.

References

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