Downscaling Seismic Data to the Meter Scale: Sampling and Marginalization

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- Background
  - Data integration
  - Upscaling issues
  - Seismic inversion data
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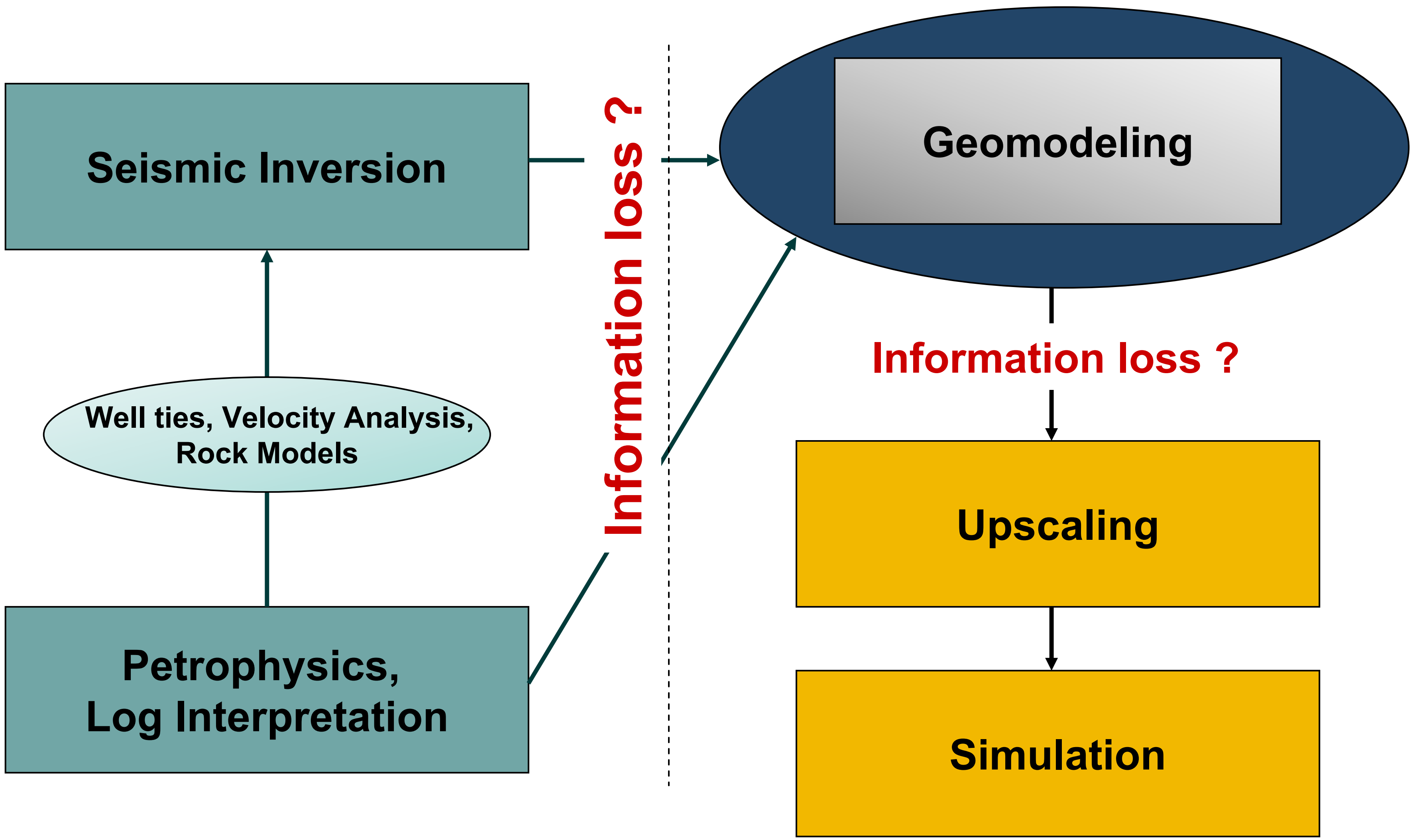
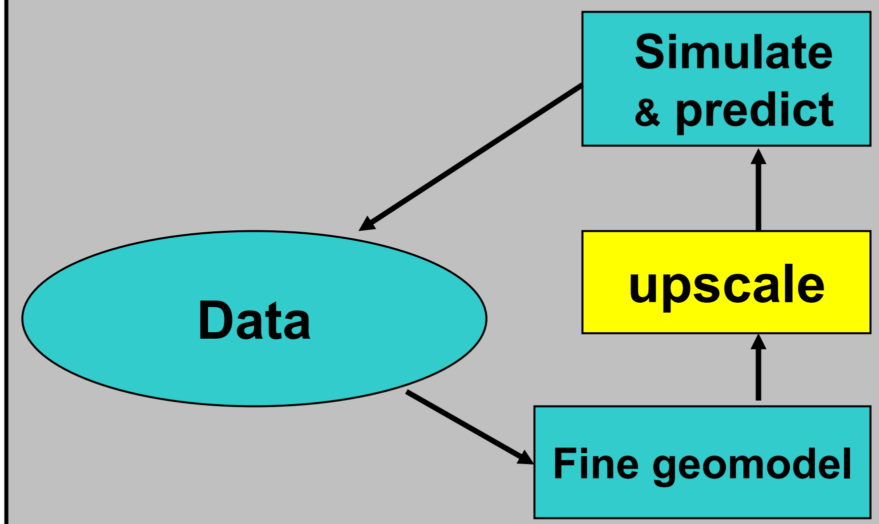
  - Auxiliary variables
  - 2D examples
- Sequential versus global simulations
  - Need for marginalization
  - 2D examples
- Conclusions

Overview



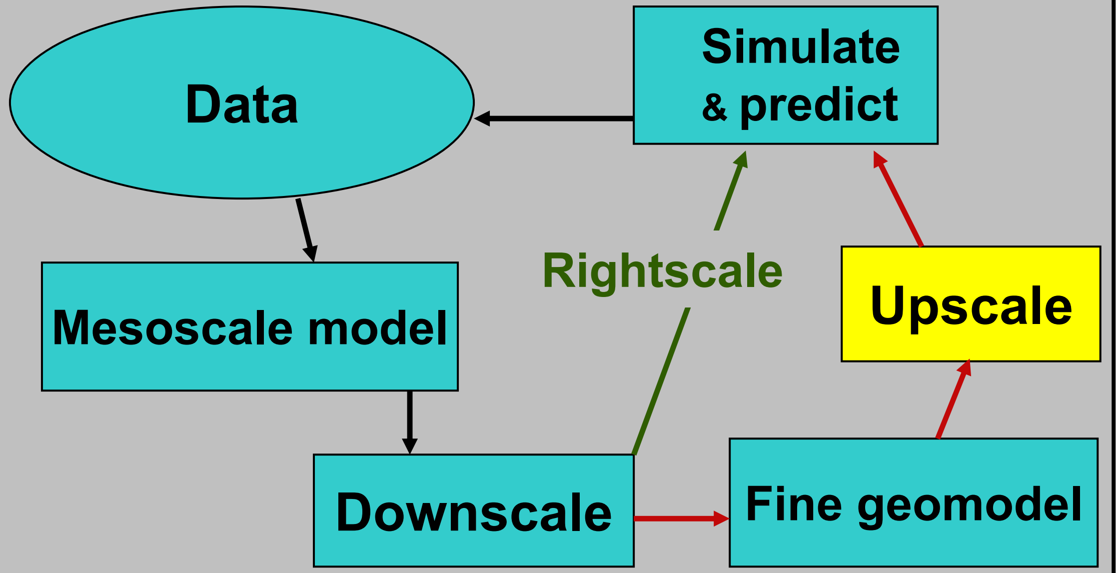
Gunning et al. 2006

Modeling Scale Issues



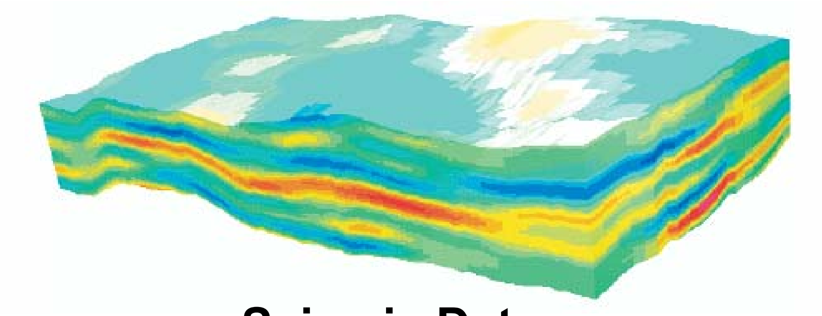
Conventional geomodeling:
very high dimensional models with horrendous inverse problems, d~10^6

Alternative model:
mesoscale mitigates horror of inverse problems, d~10^4



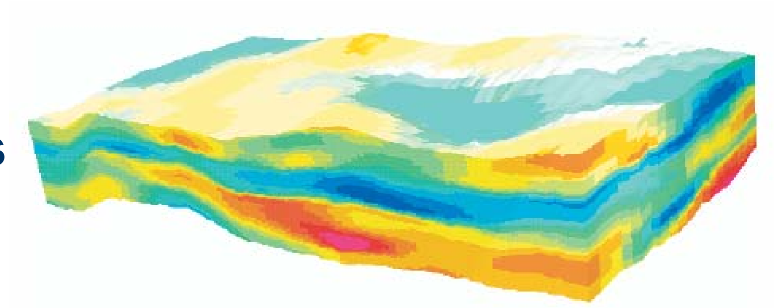
Seismic Processing using Bayesian Approach

- Seismic "calibration"
  - **Wavelet extraction**
  - **Time to depth maps**
  - **Well ties** (Gunning et al. 2003)



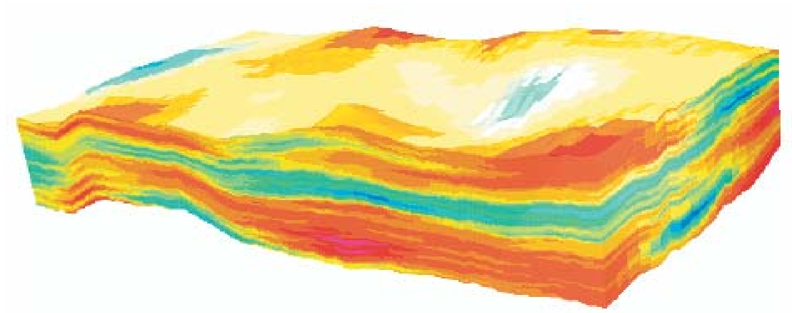
Seismic Data

- Seismic inversion
  - **Stochastic coarse scale ensembles of models** (Gunning et al. 2005)



Seismic Inversion Data (Input)

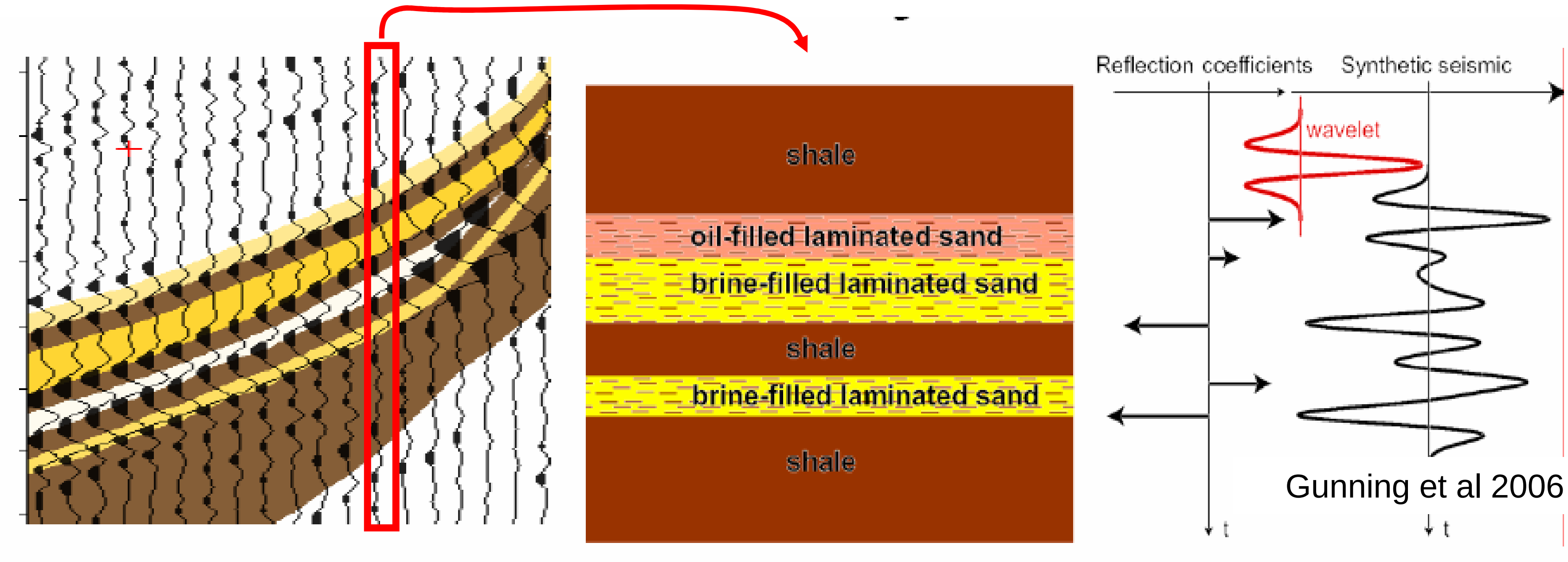
- Method to scale and integrate
  - **Enforcer for probabilistic consistency between seismic and fine-scale data**



Downscaled Data

Robinson, 2001

Wavelet and Seismic Inversion

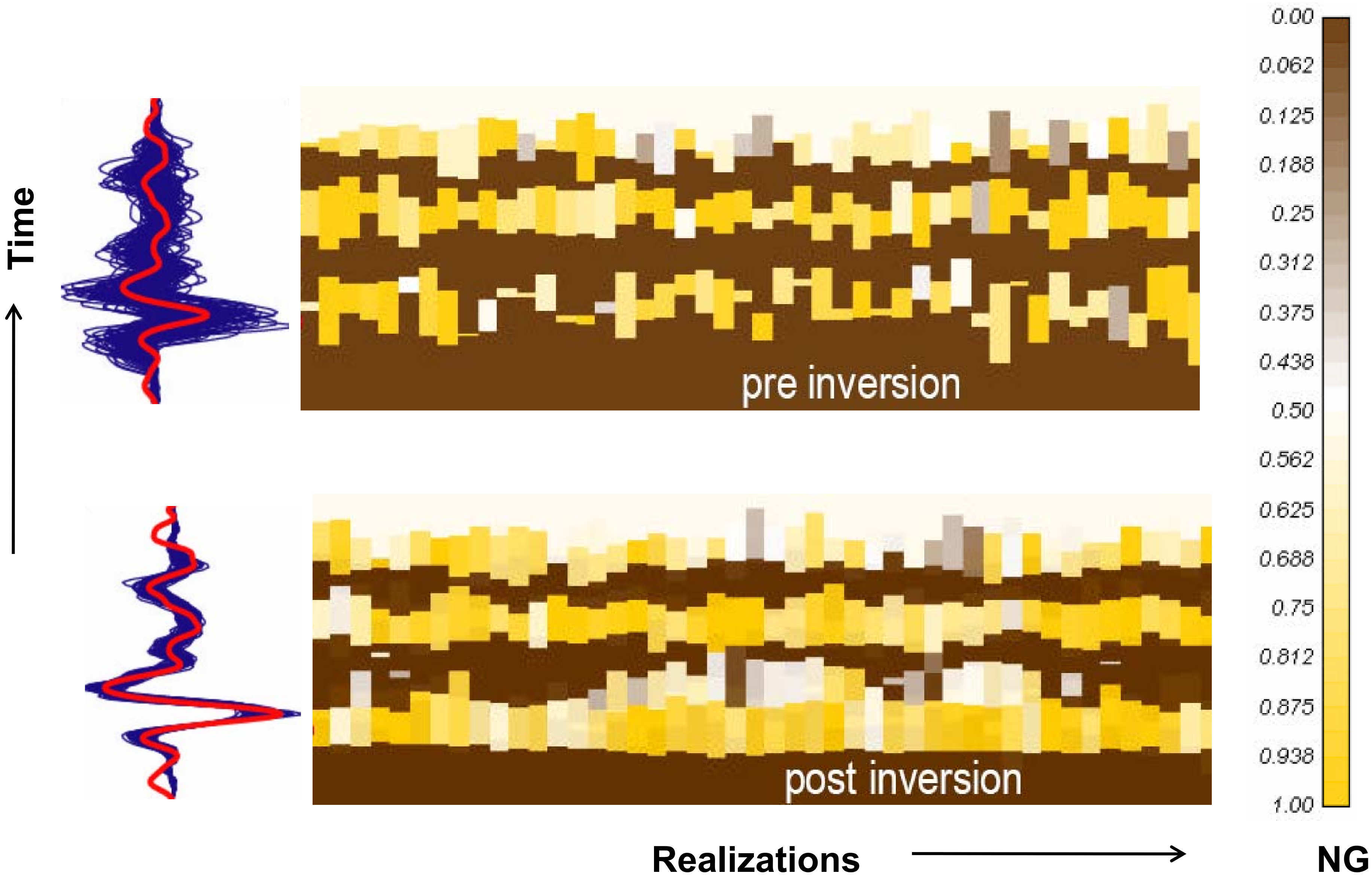


Gunning et al 2006

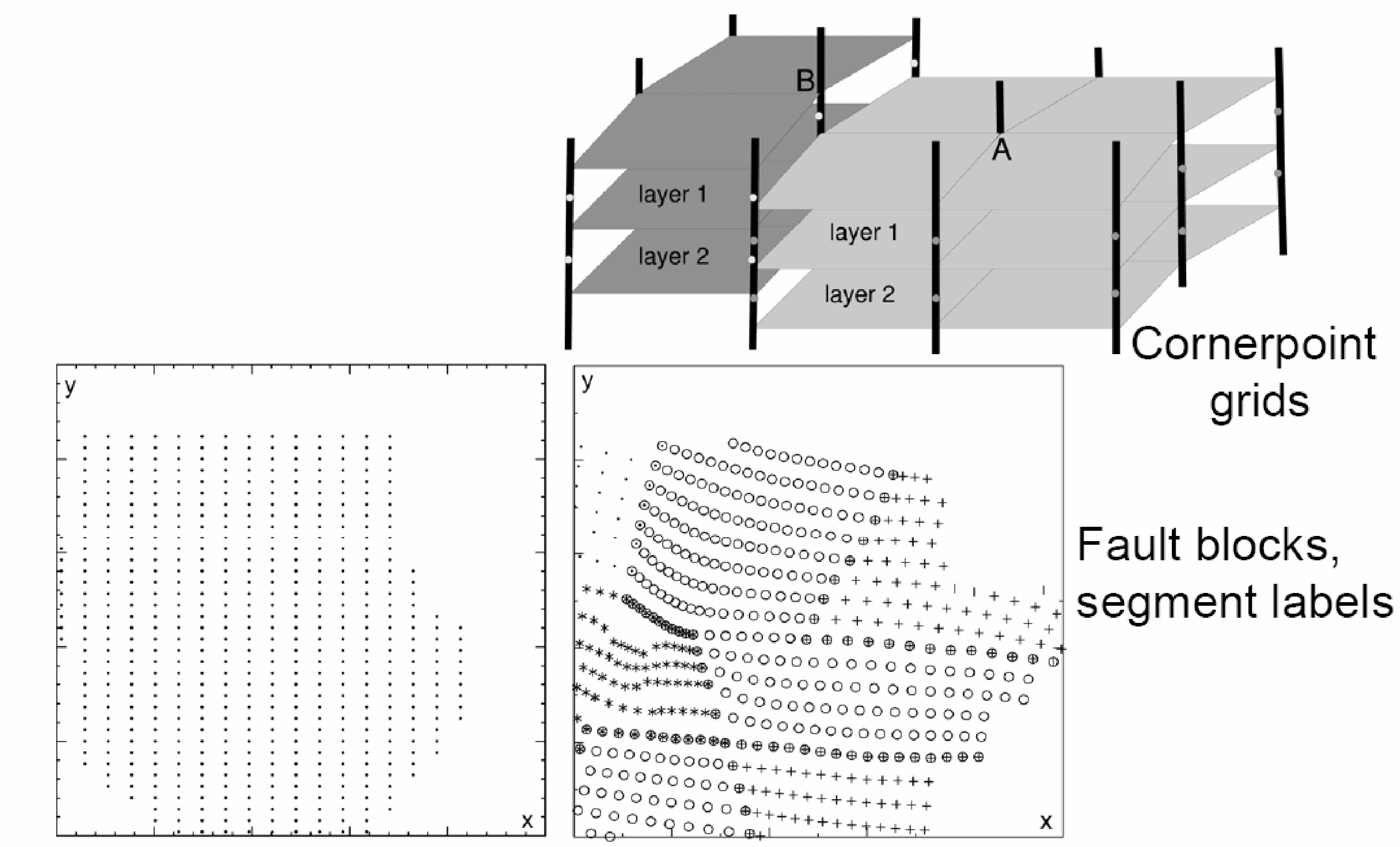
Traces At a trace Models

- Fundamental parameters
  - **Layer times**
  - **Fluid type**
  - **Rock properties in each layer**

Realizations before and after Seismic Inversion



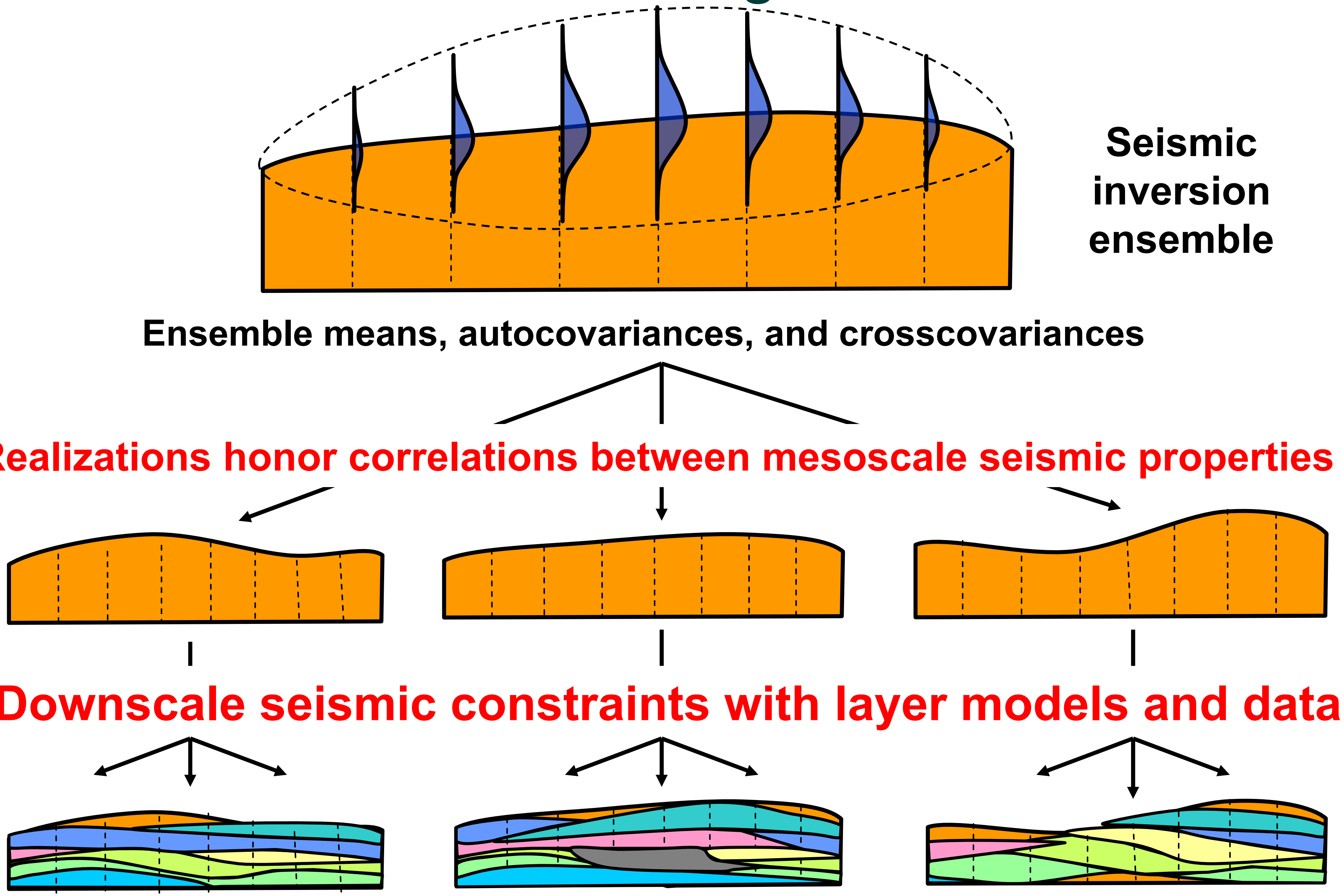
Massager: Geometric Transformation and Smoothing



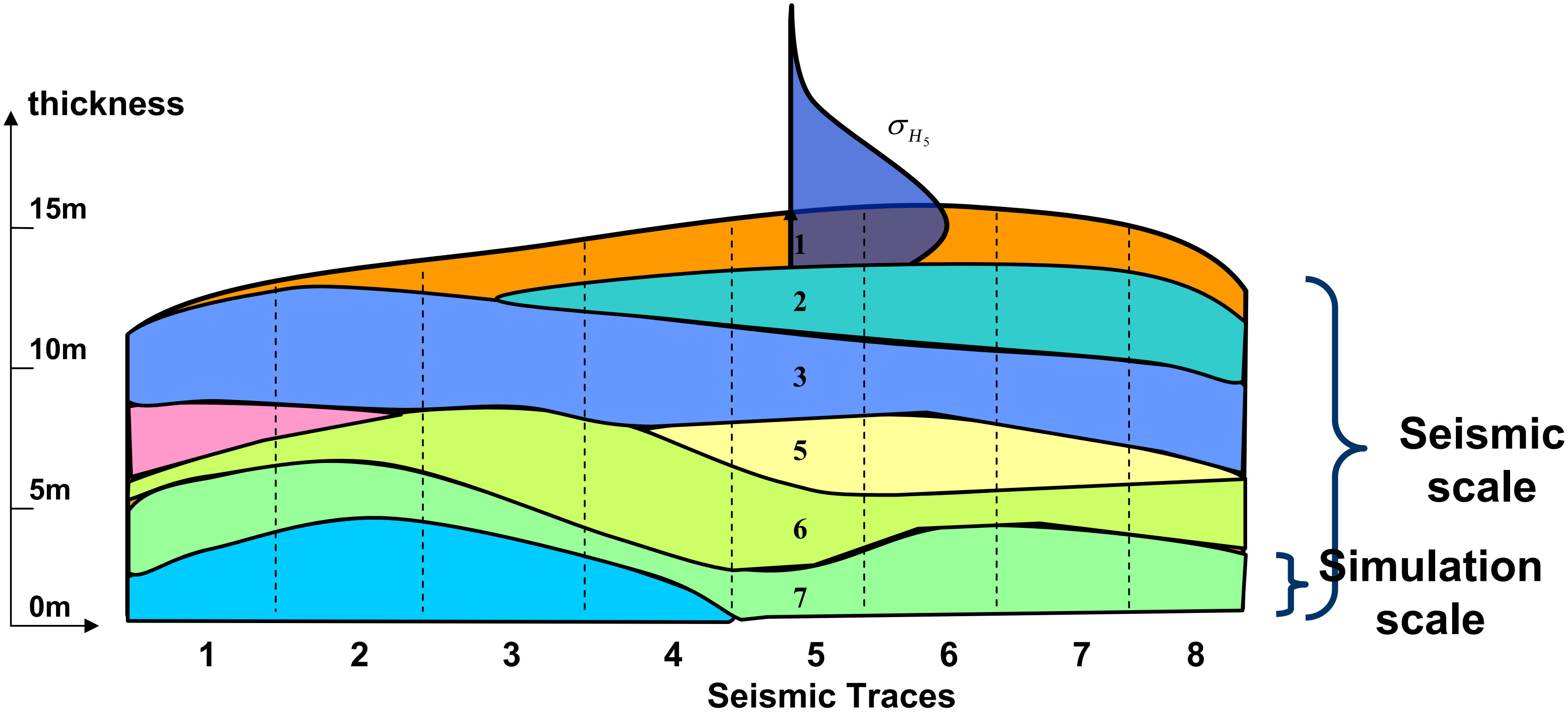
a) Seismic trace array

b) Cornerpoint grid corners, segment -coded

Exact Constraint: Working on Realizations



Inexact Constraint: Integration of Uncertain Seismic



Sum of layer thicknesses simulated should approximately match seismic thickness

Inexact Problem

MCMC Sampling with Soft Constraints

Posterior

Likelihood

Prior

$$P\left(\mathbf{t}\middle|H,\mathbf{d}_{\ell k}\right)=\frac{P\left(H\middle|\mathbf{t},\mathbf{d}_{\ell k}\right)P\left(\mathbf{t}\middle|\mathbf{d}_{\ell k}\right)}{P\left(H\middle|\mathbf{d}_{\ell k}\right)}$$

Normalizing constant

- **Prior** from variogram and nearby data $\mathbf{d}_{lk}$
- **Likelihood** from seismic mismatch
- **Posterior** by sampling many **t**
- **Normalizing** constant can be ignored

MCMC Sampling, Piecewise Gaussian Posterior

Prior

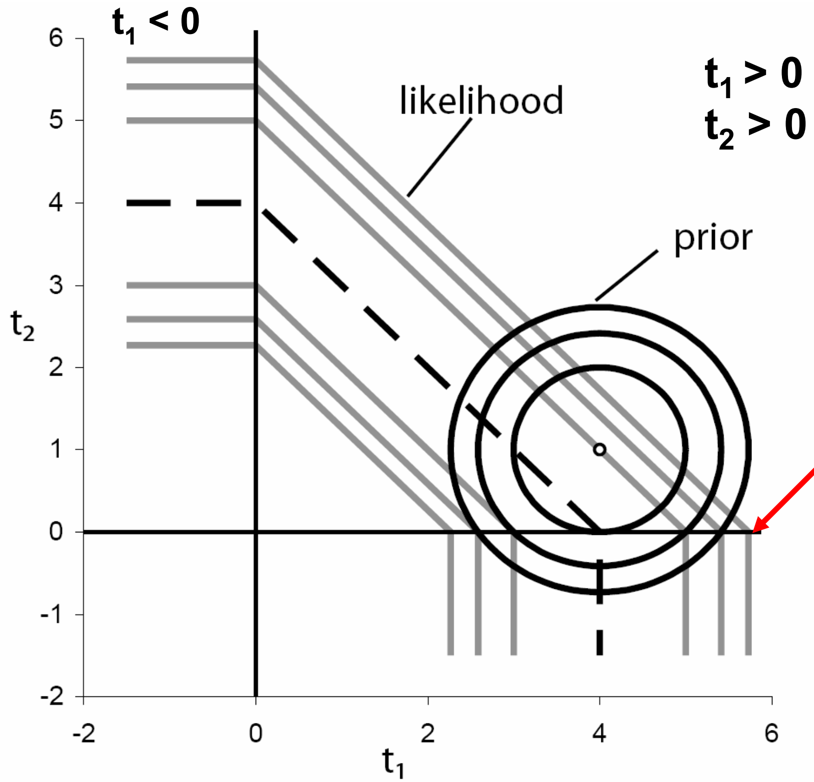
$$p\left(\mathbf{t}|\mathbf{d}_{\ell k}\right) = \frac{1}{\sqrt{(2\pi)^K |\mathbf{C}_p|}} \exp\left[-\frac{1}{2}(\mathbf{t}-\bar{\mathbf{t}})^T \mathbf{C}_p^{-1}(\mathbf{t}-\bar{\mathbf{t}})\right]$$

$$p(H|\mathbf{t},\mathbf{d}_{\ell k}) = \frac{1}{\sqrt{2\pi\sigma_H^2}} \exp\left[-\frac{(\mathbf{t}^T\mathbf{T}-\bar{H})^2}{2\sigma_H^2}\right]$$

$$\mathbf{C}_\pi = \left[\mathbf{C}_p^{-1} + \mathbf{T}\mathbf{T}^T/\sigma_H^2\right]^{-1}$$

$$T_k = \begin{cases} 0 & \text{if } t_k < 0 \\ 1 & \text{otherwise} \end{cases}$$

Likelihood



t is a Gaussian proxy for h

"Stiff" nonlinear problem

Auxiliary Variables to Sample Complicated Distributions

- Auxiliary variables generate samples from complicated distributions (Higdon, 1996)
- Lead to substantial gains in efficiency compared to standard approaches
- Inexact thickness **t** probability space is augmenting by **u**

$$\pi(\mathbf{t}, \mathbf{u}) = \pi(\mathbf{t})\pi(\mathbf{u}|\mathbf{t})$$

$$\pi\left(u_k = 1 | t_k\right) = \begin{cases} 1 - \frac{1}{2 + t_k/\sigma_{\pi k}} & \text{if } t_k \geq 0 \\ \frac{1}{2 - t_k/\sigma_{\pi k}} & \text{otherwise} \end{cases}$$

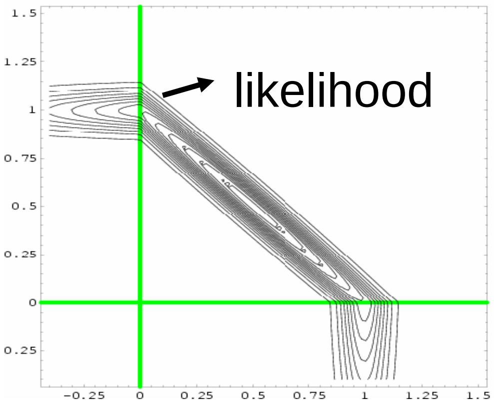
- Define auxiliary variable $u_k = \{0, 1\}$ as indicator of truncation, 1 for $t_k > 0$
- u_k is updated with a Gibbs step and Metropolis step to update the t_k

Data Augmentation to Handle Bends in the Posterior

- Reversible MCMC hopping scheme that adjust to the proposals to the shape of local posterior

$$\mathbf{C}_\pi = \left[\mathbf{C}_p^{-1} + \mathbf{T}\mathbf{T}^T/\sigma_H^2\right]^{-1}$$

$$\mathbf{C}_t = s\mathbf{C}_\pi \quad \text{where } s = \frac{5.76}{K}$$

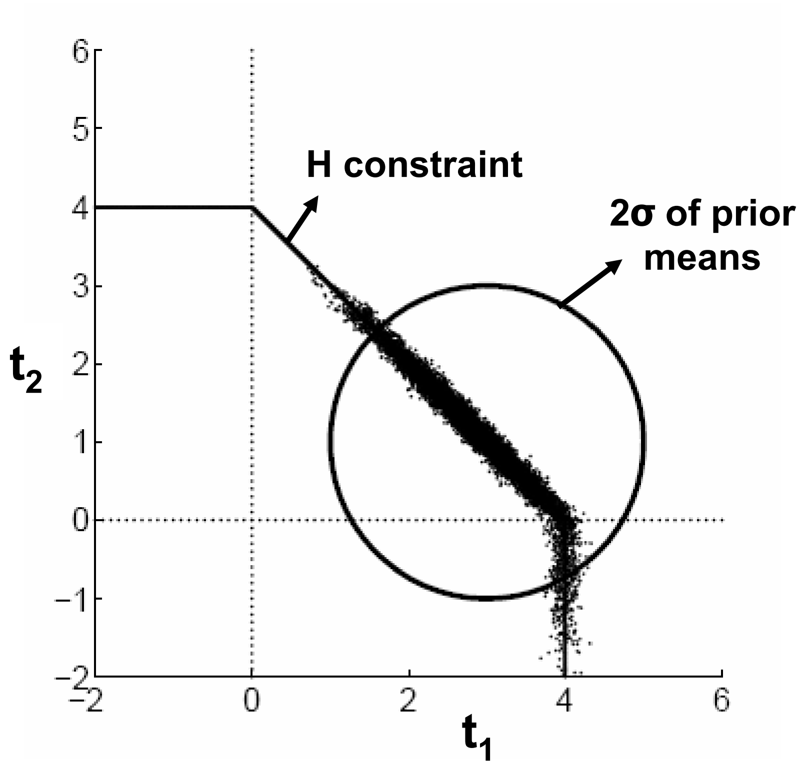


Gelman 2003; Roberts

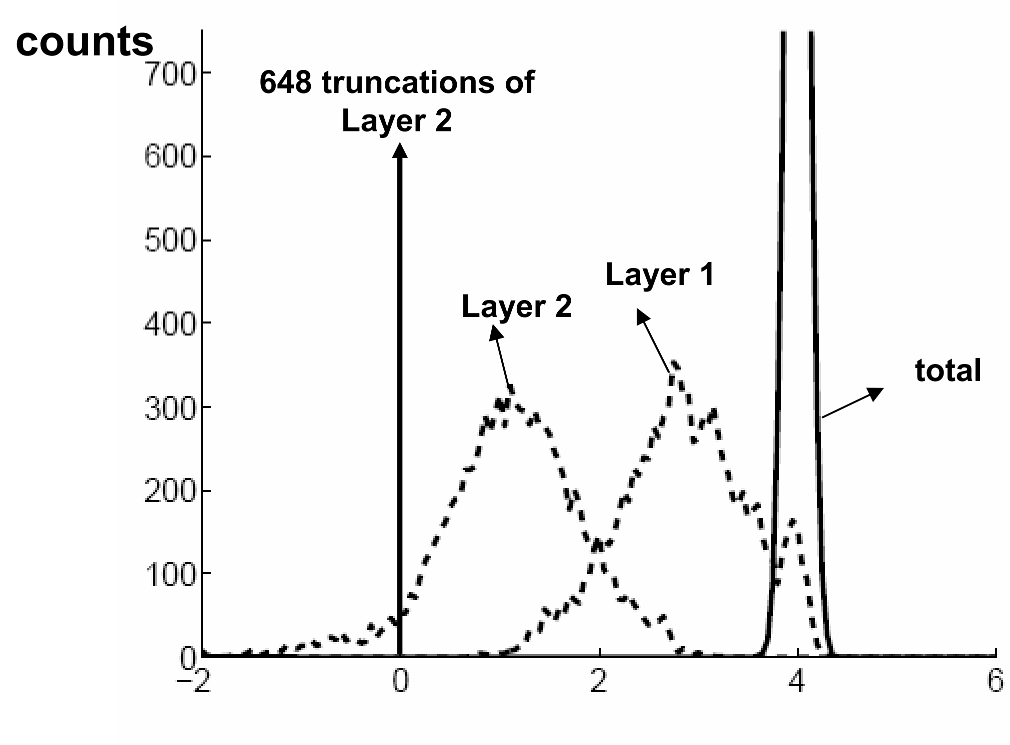
- Metropolis transition probability for t includes thickness and auxiliary terms

$$\alpha = \min\left(1, \frac{\pi\left(\mathbf{t}'|H, \mathbf{d}_{\ell k}\right)\prod_{k=1}^{K}\pi(u_k|t'_k)}{\pi\left(\mathbf{t}|H, \mathbf{d}_{\ell k}\right)\prod_{k=1}^{K}\pi(u_k|t_k)}\right)$$

Pinching Layer with Consistent Tight Sum Constraint



Scattergram, N = 8,000



Histogram of t

Likelihood: $\overline{H} = 4\text{ m},\ \sigma_H = 0.1\text{ m}$

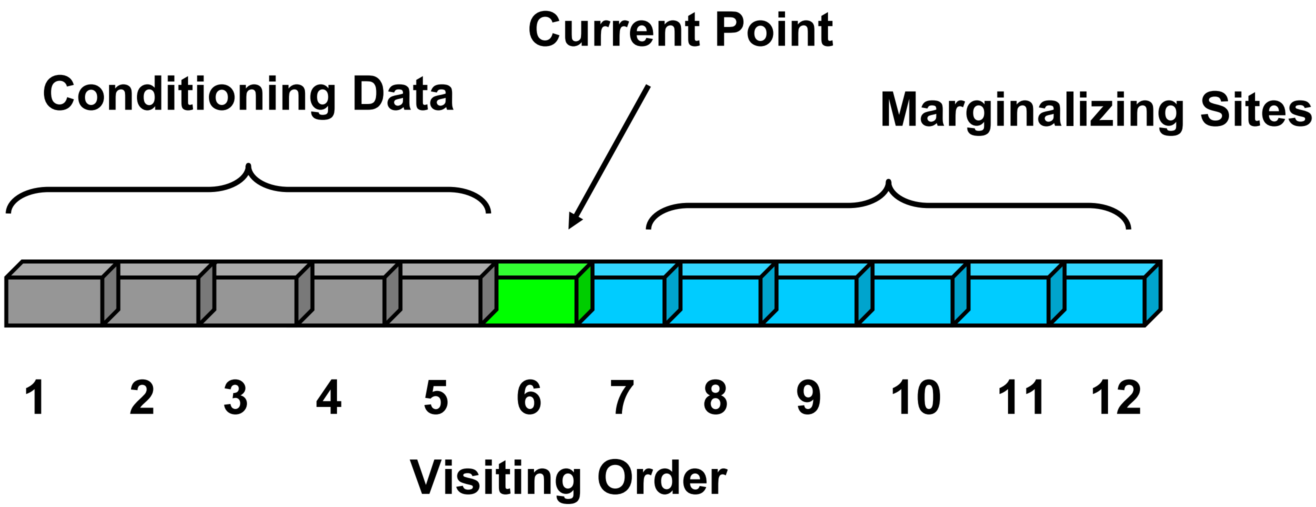
Prior: $\bar{\mathbf{t}} = (3\text{ m},\ 1\text{ m}),\ \sigma_t = 1\text{ m}$

Marginals

Comparison of Global and Sequential methods

The posterior can be decomposed in to product of marginals for sequential simulation

$$\pi(\mathbf{t}) = \pi(\mathbf{t}_1)\pi(\mathbf{t}_2|\mathbf{t}_1)\ldots\pi(\mathbf{t}_I|\mathbf{t}_1\ldots\mathbf{t}_{I-1})$$



Marginal needs to integrate un-simulated sites

$$\pi(\mathbf{t}_6|\mathbf{t}_1\ldots\mathbf{t}_5,\mathbf{H}) = \int_{-\infty}^{\infty}\prod_{j=5}^{12} L\left(\mathrm{H}_j|\mathbf{t}_j\right) p\left(\mathbf{t}|\mathbf{t}_1\ldots\mathbf{t}_5\right) \mathrm{d}\mathbf{t}_7\ldots\mathrm{d}\mathbf{t}_{12}$$

Marginals for Sequential Gaussian Simulation

If all t are Gaussian functions then also marginals are to be computed at each trace

$$\pi(\mathbf{t}) = \pi(\mathbf{t_1})\pi(\mathbf{t_2}|\mathbf{t_1})\ldots\pi(\mathbf{t}_I|\mathbf{t}_1\ldots\mathbf{t}_{I-1})$$

Marginal at the first trace

$$\pi(t_1|\mathbf{d}) = \int \pi(\mathbf{t}|\mathrm{d})\mathrm{d}\mathbf{t}_*$$

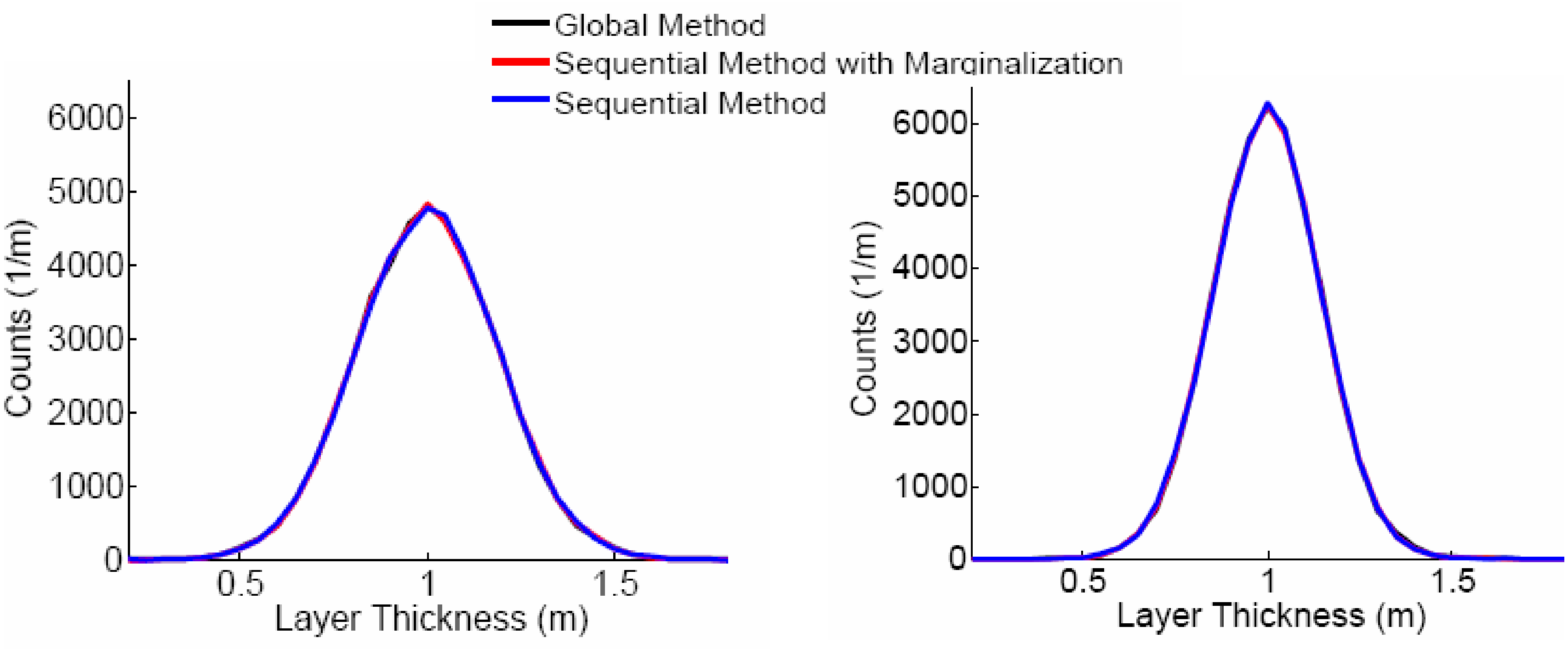
where $\mathbf{t}_* = (\mathrm{t}_2\ldots\mathrm{t}_{12})$

$$\pi(\mathrm{t}_1|\mathbf{d}) \propto \int \exp\left[-\frac{1}{2}\begin{pmatrix} \mathrm{t}_1-\mu_1 \\ \mathbf{t}_*-\boldsymbol{\mu}_* \end{pmatrix}^{\mathrm{T}} \begin{pmatrix} \mathrm{C}_{11} & \mathbf{C}_{1*} \\ \mathbf{C}_{*1} & \mathbf{C}_{**} \end{pmatrix}^{-1} \begin{pmatrix} \mathrm{t}_1-\mu_1 \\ \mathbf{t}_*-\boldsymbol{\mu}_* \end{pmatrix}\right]\mathrm{d}\mathbf{t}_*$$

$$\propto \exp\left[-\frac{1}{2}(\mathrm{t}_1-\mu_1)\mathrm{C}_{11}^{-1}(\mathrm{t}_1-\mu_1)\right]$$

Marginal doesn't depend on un-simulated sites for pure multi-Gaussian distributions

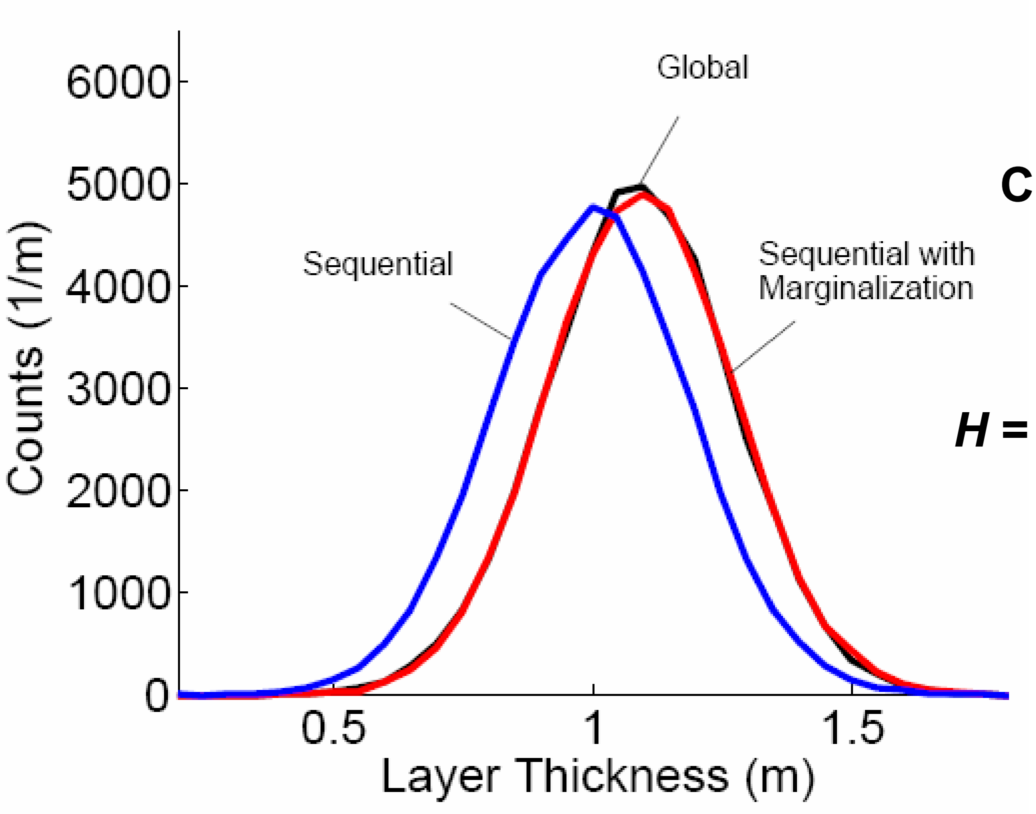
Comparison of Global and Sequential Methods
Verify: Marginalization not Needed if Weakly Correlated

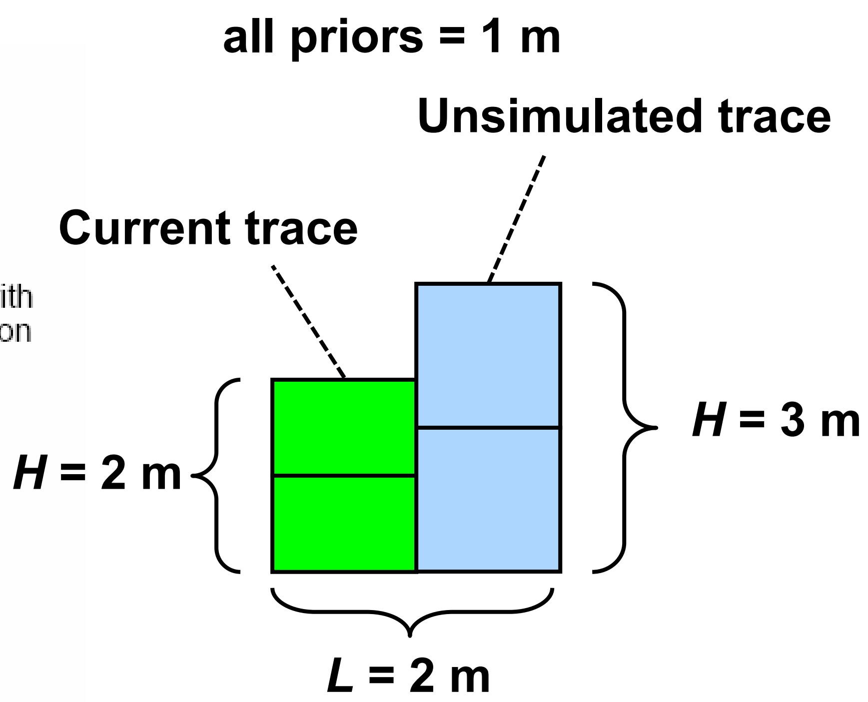


(a) Weak seismic data, Weak correlation **(a) Strong seismic data, Weak correlation**

- **<u>S</u>equential simulation with <u>M</u>arginals (SM)**
  - **Marginal is integrable assuming linearity in unsimulated points and "weak correlations"**
- **<u>S</u>equential <u>S</u>imulation (SS)**
  - **Heavier approximation of no lateral correlation gives SS**
- **Compared to a rigorous <u>G</u>lobal <u>M</u>ethod (GM) (Using exhaustive MCMC)**

Comparison of Global and Sequential Methods
Success: Integrates Surrounding Constraints

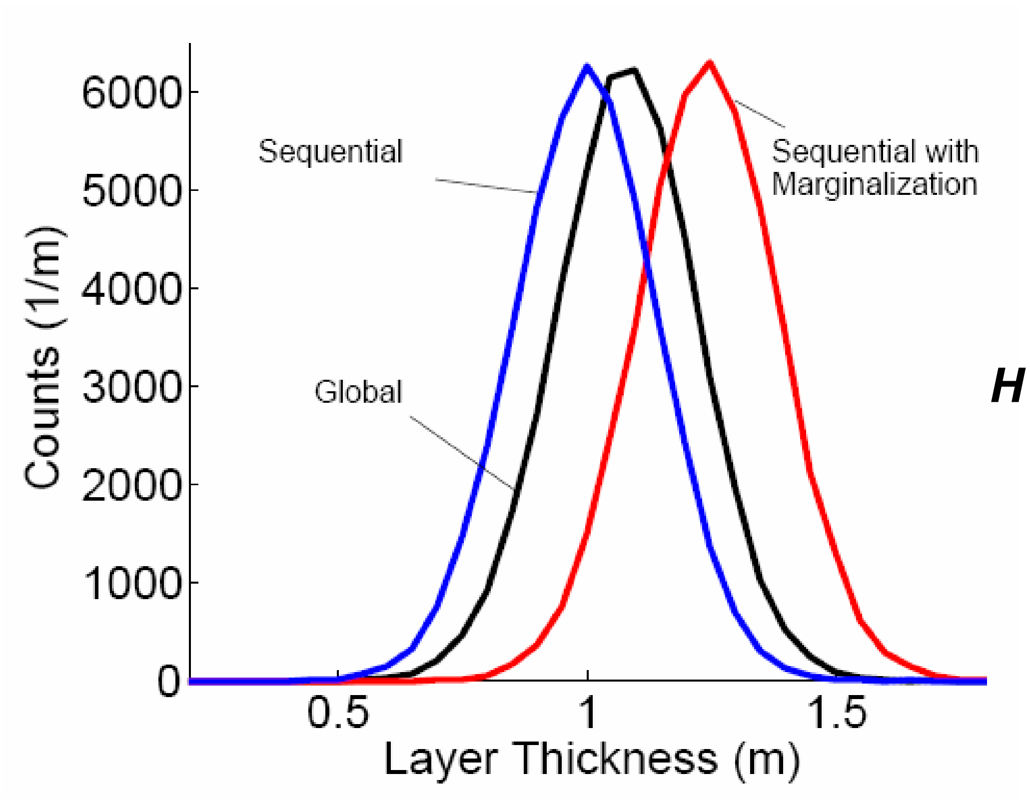


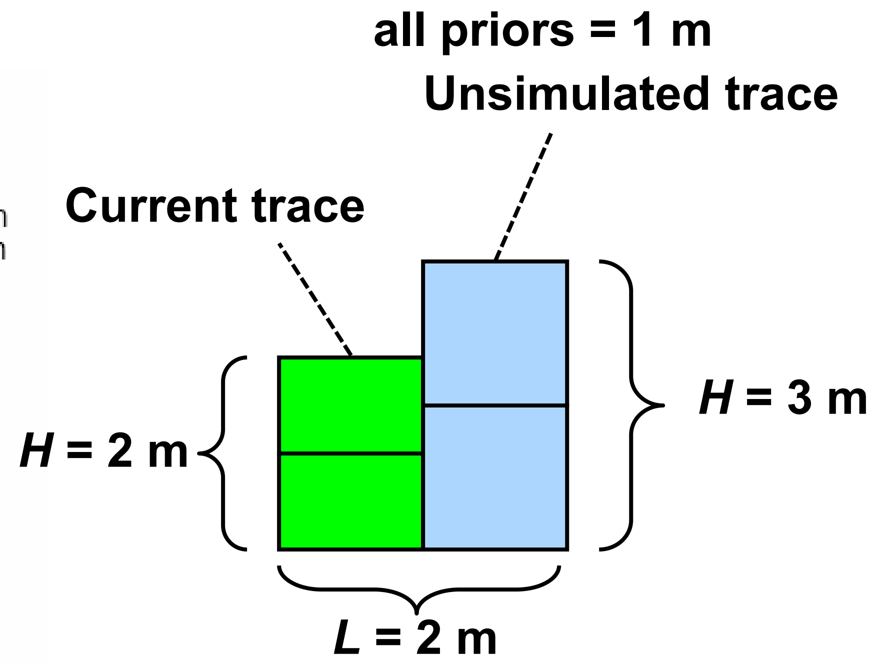


$$\frac{\sigma_H}{H} = 0.25 \qquad \frac{\lambda}{L} = 2.0$$

Strong correlation, constraint locally weak, marginalization improves simulation

Comparison of Global and Sequential Methods
Failure: Inconsistent Data





$$\frac{\lambda}{L} = 2.0 \qquad \frac{\sigma_H}{H} = 0.025$$

Strong correlation, constraint locally strong:
poor results if nearby mean seismic thicknesses are not consistent

Conclusions

- Framework for multi-property data integration accounting for the scale and precision of different data types
- Poor mixing of MCMC samplers for exotic-shaped posteriors much improved using auxiliary variables
- Sequential methods: full distributions should be marginalized at current trace in certain conditions
  - Especially if correlation is strong ($\lambda/L > 0.5$)
  - Proposed marginalization may fail if $\frac{\bar{H} - \bar{H}_{\mathrm{Marg}}}{\sigma_H}$ is not small
- Sequential methods appear adequate if lateral correlations and updating constraints are weak

Acknowledgements

BHP-Billiton for funding this research

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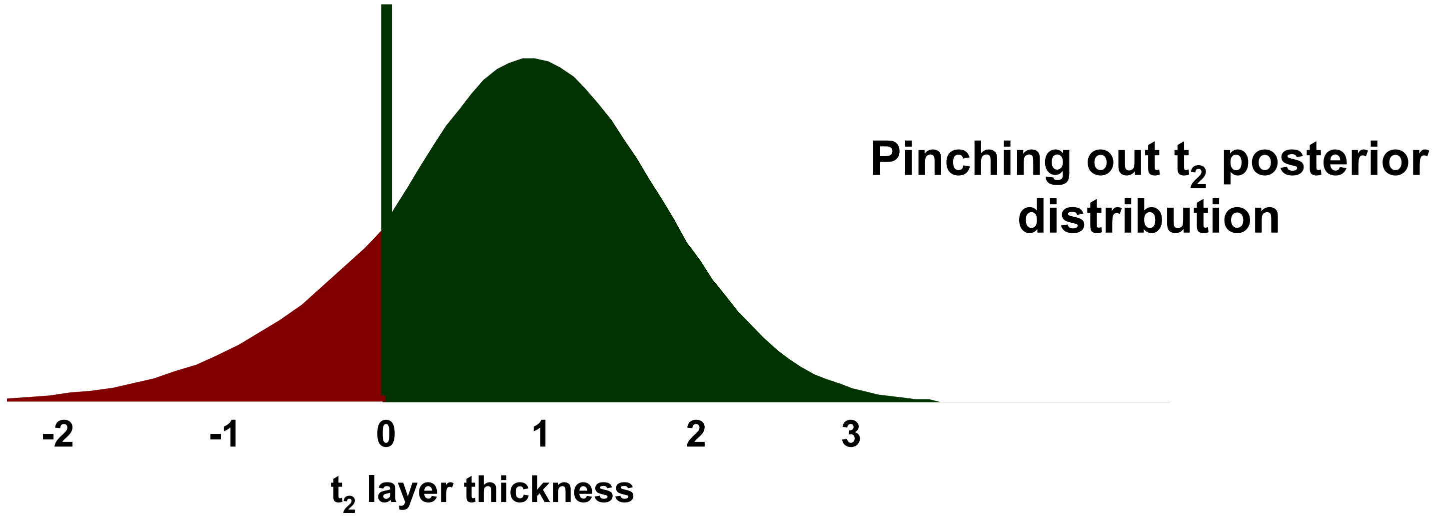
Backups

Handling Pinchouts



- A Gaussian model is efficient and simple, but some of the *t* (proxies for *h*) are negative
- Set geomodel thickness $h = 0$ if $t<0$

Comparison of Global and Sequential methods

The posterior as a product of marginals for sequential simulation

$$\pi(\mathbf{t}) = \pi(\mathbf{t}_1)\pi(\mathbf{t}_2|\mathbf{t}_1)\ldots\pi(\mathbf{t}_I|\mathbf{t}_1\ldots\mathbf{t}_{I-1})$$

Marginal integrates unsimulated data

$$\pi(\mathrm{t}_i|\mathrm{t}_1\ldots\mathrm{t}_{i-1},\mathrm{d}) = \int_{-\infty}^{\infty}\prod_{j=i+1}^{I} L\left(\mathbf{H}_j|\mathrm{t}_j,\mathrm{d}\right)p\left(\mathrm{t}|\mathrm{d}\right)\mathrm{d}\mathrm{t}_{i+1}\ldots\mathrm{d}\mathrm{t}_I$$

Analytically integrable if we assume
linearized constraints at unsimulated points and "weak correlations"

$$\pi\left(\mathbf{t}_1|\mathbf{H}\right)\propto e^{-\frac{(f(\mathbf{t}_1)-\mathbf{H}_1)^2}{2\sigma^2_{\mathbf{H}_1}}}\int_{-\infty}^{\infty}e^{-\frac{1}{2}(\mathbf{X}_2\mathbf{t}_2-\mathbf{H}_2)^T\mathbf{C}^{-1}_{\mathbf{H}_2}(\mathbf{X}\mathbf{t}_2-\mathbf{H}_2)}p\left(\mathbf{t}_1,\mathbf{t}_2\right)\mathrm{d}\mathrm{t}_2$$

No lateral correlation -->
standard sequential simulation (without marginalization)

$$\pi\left(\mathbf{t}_1|\mathbf{H}\right)\propto e^{-\frac{(f(\mathbf{t}_1)-\mathbf{H}_1)^2}{2\sigma^2_{\mathbf{H}_1}}}p\left(\mathbf{t}_1\right)$$

Problem Formulation in Bayesian Form

Likelihood

Posterior

Prior

$$\pi\left(\mathbf{t},\phi \middle| H,\Phi,\mathbf{d}_{\ell k}\right) = \frac{p\left(H,\Phi \middle| \mathbf{t},\phi,\mathbf{d}_{\ell k}\right) p\left(\mathbf{t},\phi \middle| \mathbf{d}_{\ell k}\right)}{p\left(H,\Phi \middle| \mathbf{d}_{\ell k}\right)}$$

Normalizing constant

- **Prior** from variogram and nearby data $\mathbf{d}_{lk}$
- **Likelihood** from seismic mismatch
- **Posterior** by sampling many $\mathbf{t}$
- **Normalizing** constant can be ignored

Expressions and Visual Clues of Bayesian Terms

Prior

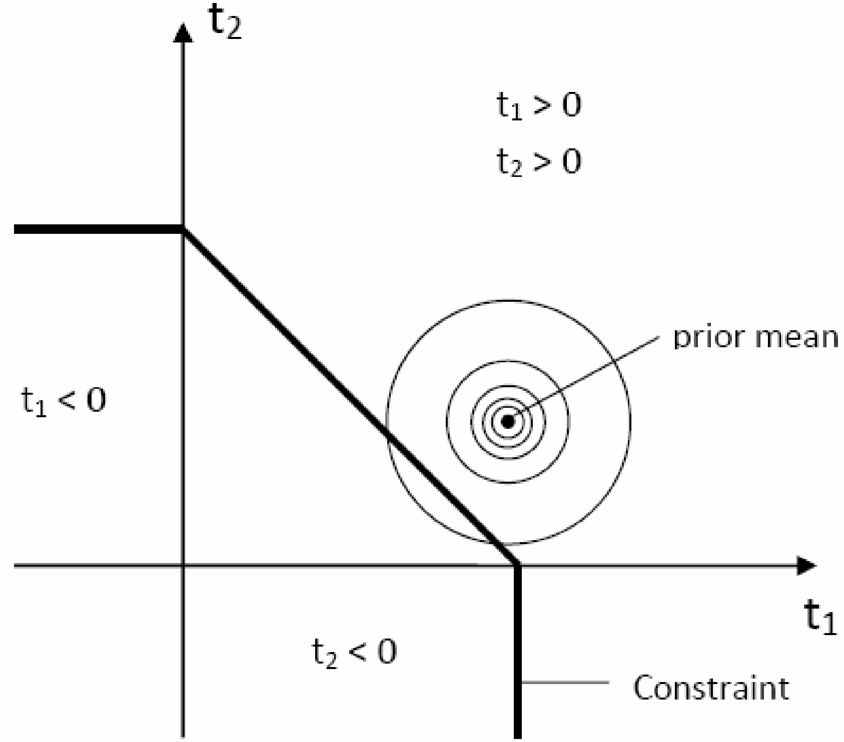
$$p\left(\mathbf{t}|\mathbf{d}_{\ell k}\right) = \frac{1}{\sqrt{(2\pi)^K \left|\mathbf{C}_p\right|}} \exp\left[-\frac{1}{2}(\mathbf{t}-\bar{\mathbf{t}})^T \mathbf{C}_p^{-1}(\mathbf{t}-\bar{\mathbf{t}})\right]$$

$$p\left(H|\mathbf{t},\mathbf{d}_{\ell k}\right) \propto \begin{cases} 0 & \text{,constraints not satisfied} \\ 1 & \text{,otherwise} \end{cases}$$

Likelihood

$$\pi\left(\mathbf{t}|H,\mathbf{d}_{\ell k}\right) \propto \begin{cases} 0 \\ p\left(\mathbf{t}|\mathbf{d}_{\ell k}\right) \end{cases}$$

Posterior



MCMC Sampling with Exact Constraints

- **Basis orthogonal to $\mathbf{u} = (1, 1, \ldots, 1)$**
- **Random walk in R where**

$$\boldsymbol{\tau} = (\delta, \mathbf{r})$$

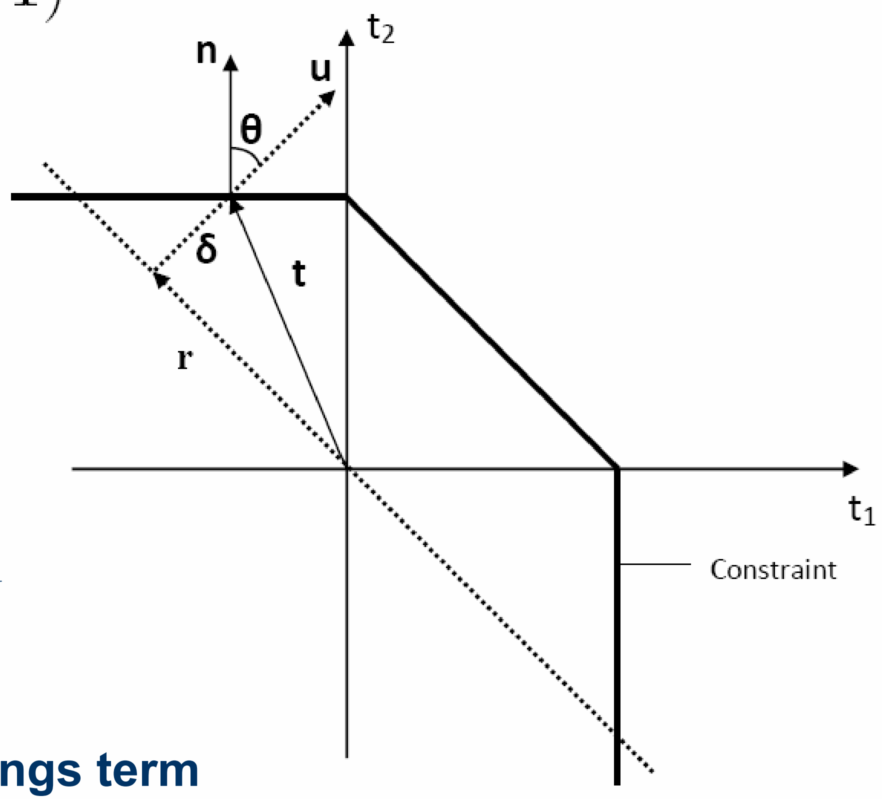
- **Obtain δ by solving**

$$\sum_{j=1}^{K} \max(0, \mathbf{t}_j(\delta, \mathbf{r})) = H$$

- **Transform coordinates using matrix $\mathbf{U}$**

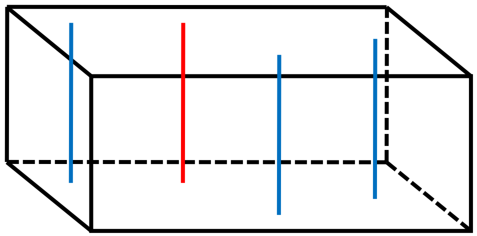
$$\mathbf{t} = \mathbf{U}.\boldsymbol{\tau}$$

- **Jacobian included in Metropolis-Hastings term**

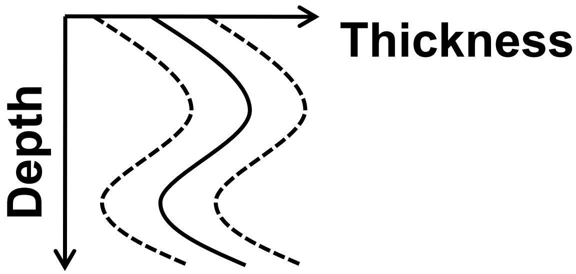


$$\tilde{\pi}(\mathbf{t}) = \pi(\mathbf{t})|\sec(\theta)| = \pi(\mathbf{t})\frac{1}{\mathbf{u}.\mathbf{n}(\mathbf{t})}$$

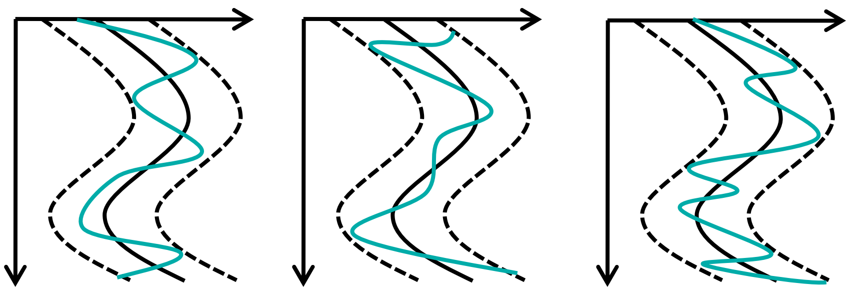
Sequential MCMC



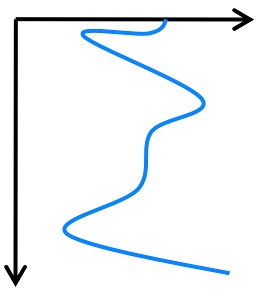
Generate path and select an estimation location randomly (red line) from within the 3D volume



Krige the well data (blue line) to derive the mean (solid line) and variance (dashed line) of the log data at the estimation location

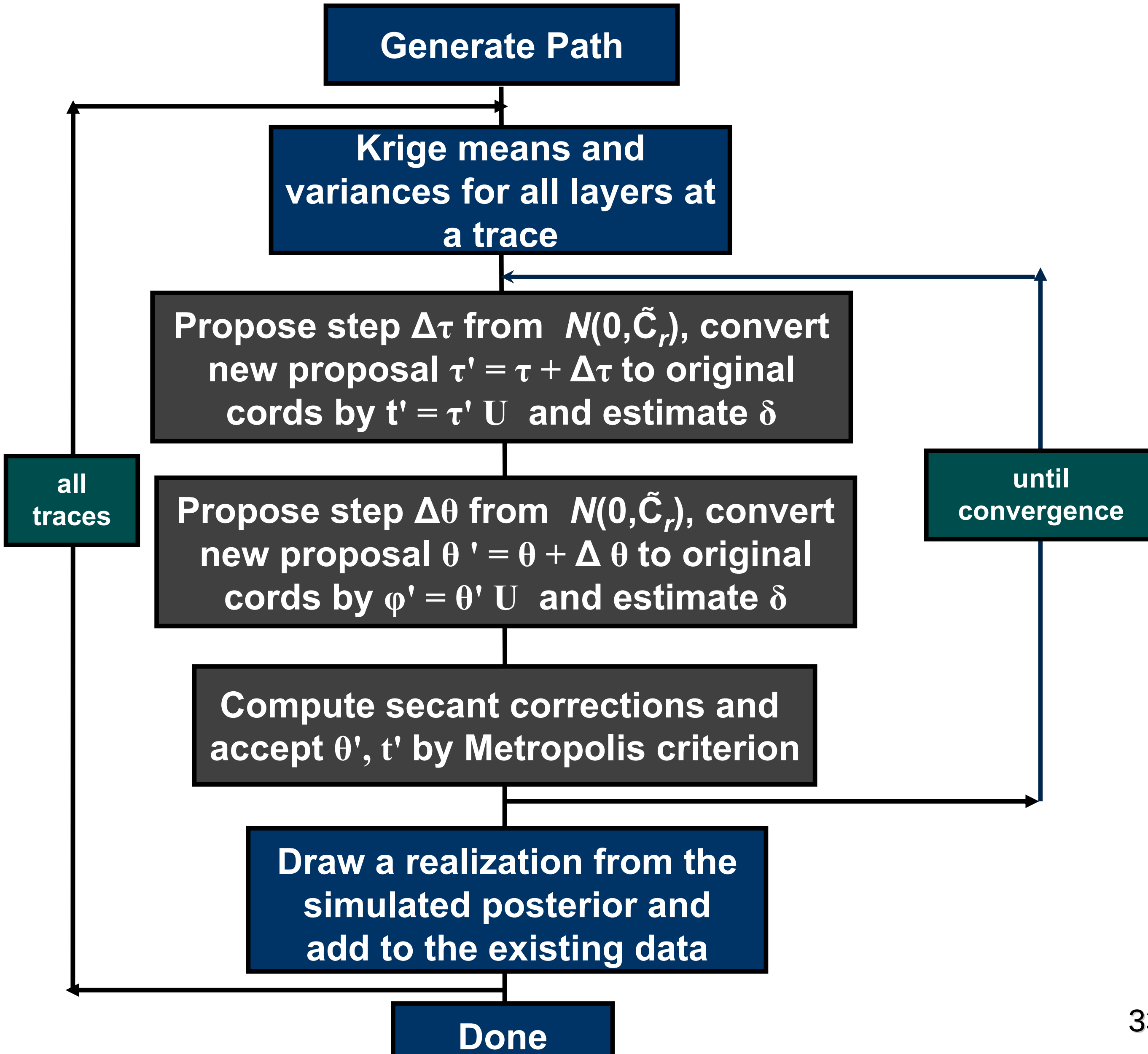


Propose many equiprobable thickness and porosity values that match seismic inversion data

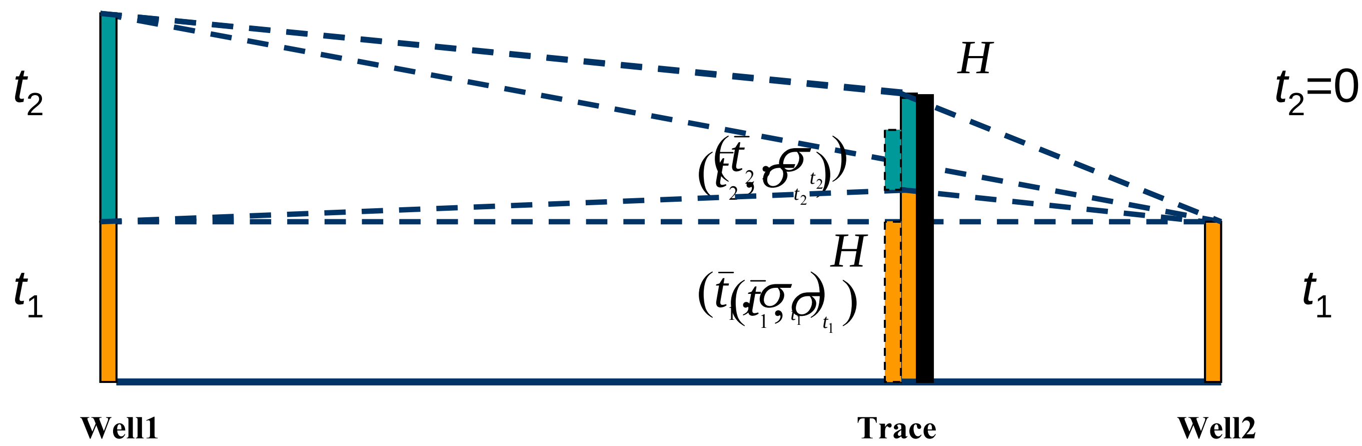


Randomly retain one as the solution and repeat the process at all the traces

Sequential TG-MCMC

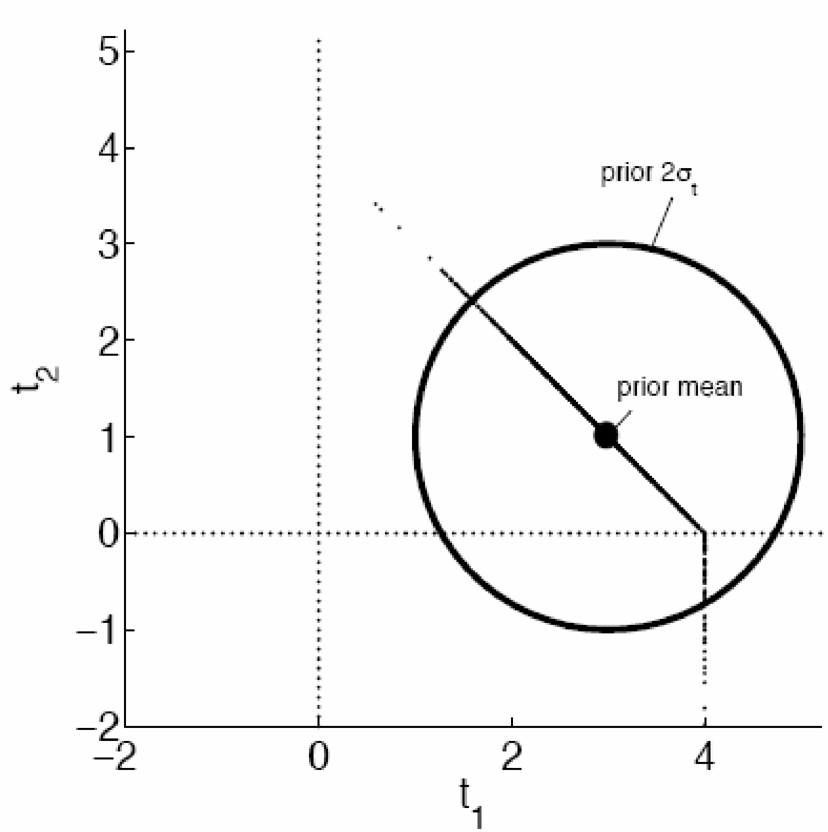


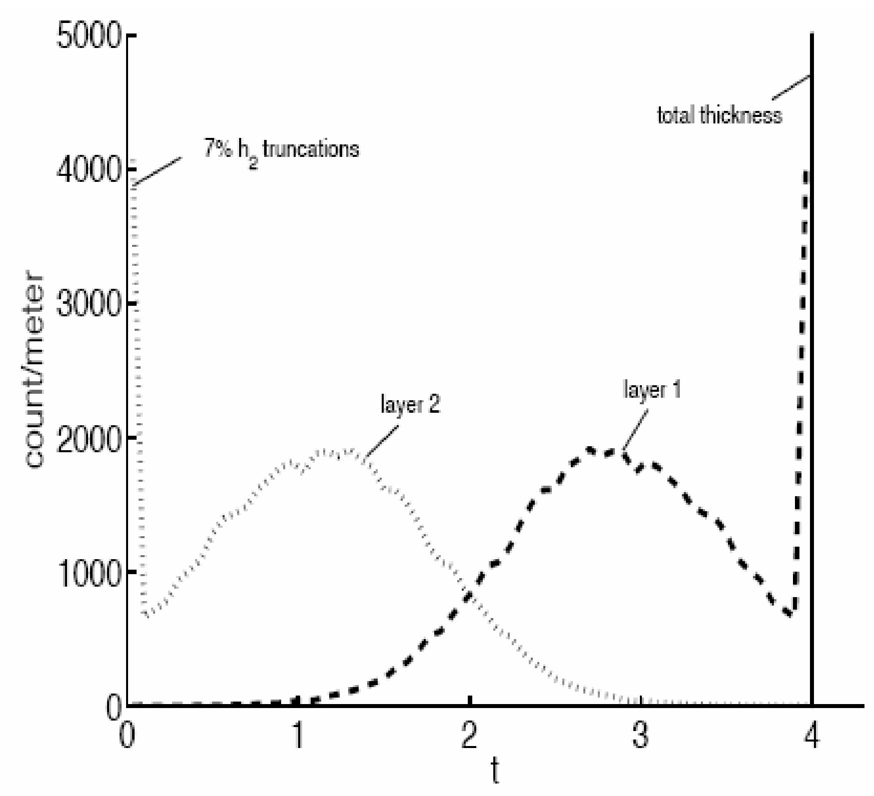
A Simple Two layer Case



- Bayes reconciles seismic and well/continuity data
- Simulation retrieves the complete distribution, not just the most likely combination

Pinching Layer with Consistent Thickness Sum Constraint

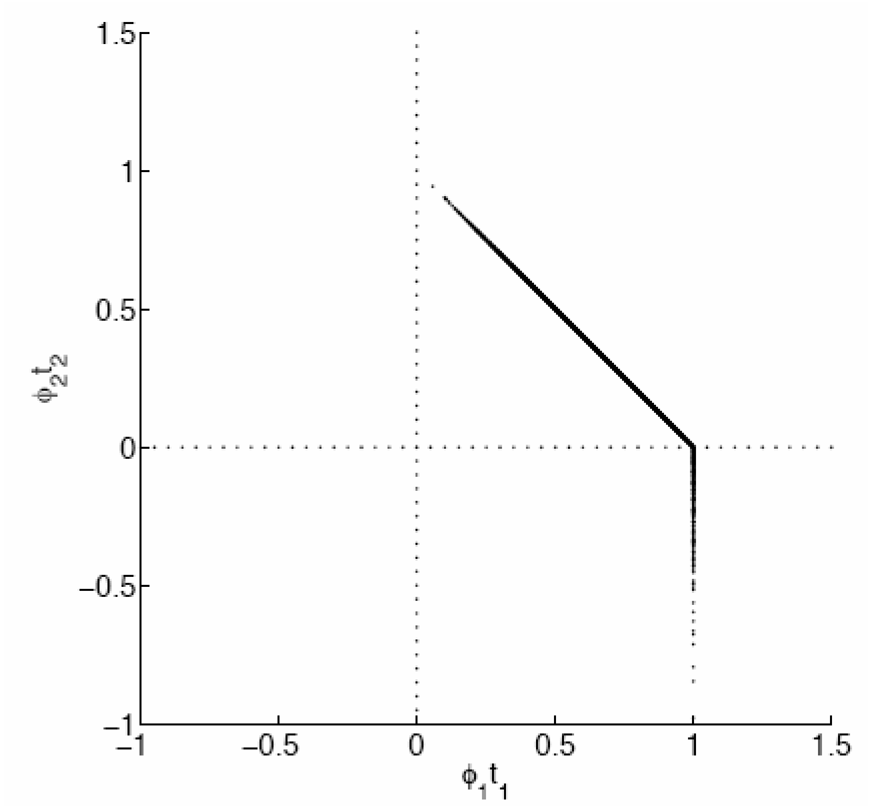


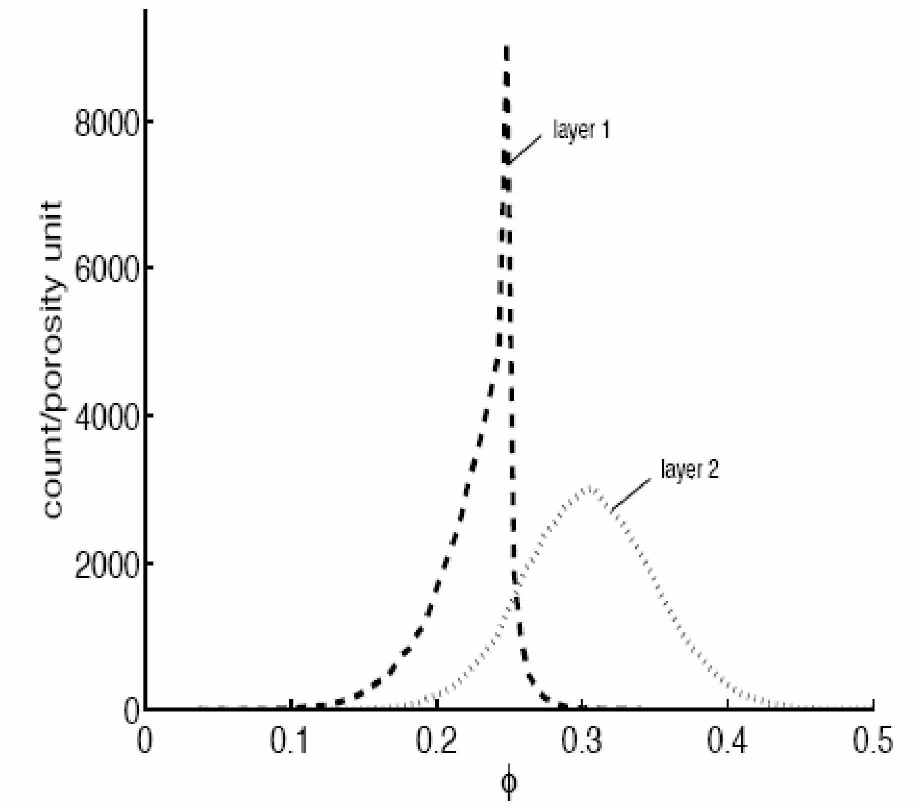


Likelihood: $H = 4\text{m}$

Prior: $\bar{\mathbf{t}} = (3\text{ m}, 1\text{ m}), \sigma_t = 1\text{ m}$

Samples Honoring Porosity Constraint

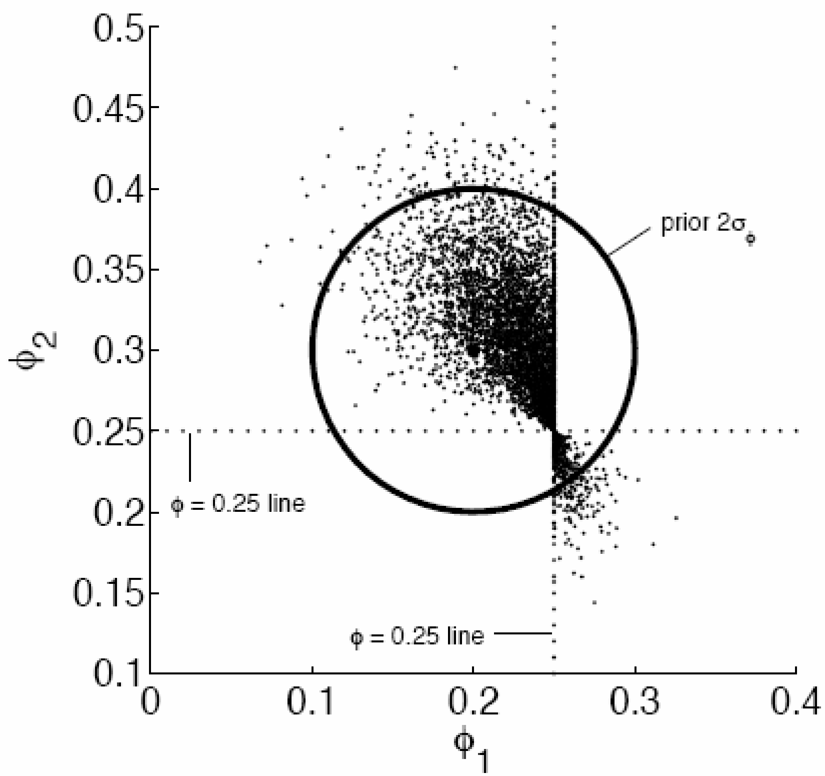


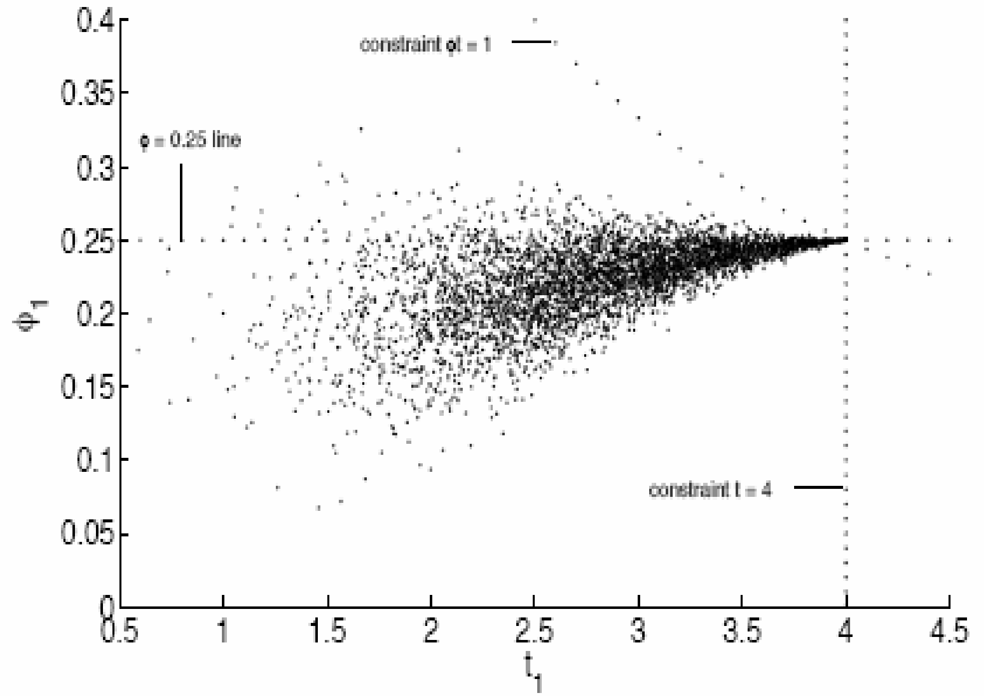


$$\Phi = 0.25 \Rightarrow \Phi H = 1.0$$

$$\bar{\phi} = (0.2, 0.3), \sigma_\phi = 0.05$$

Cross Plots of Thickness and Porosity





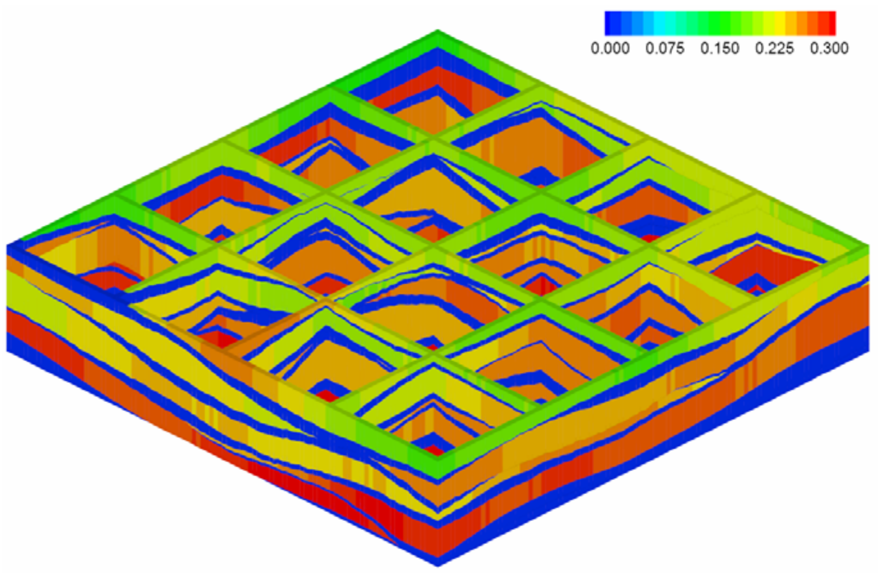
$$\Phi = 0.25 \Rightarrow \Phi H = 1.0$$

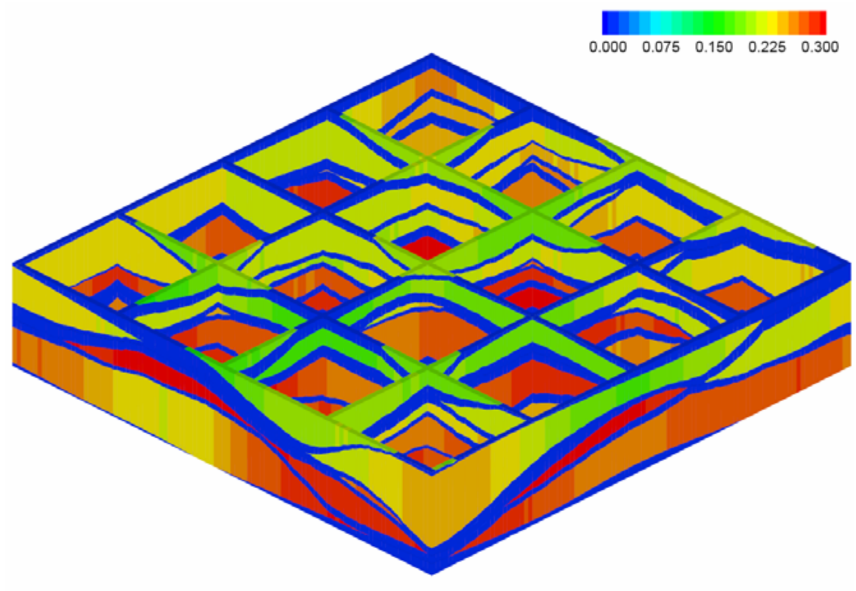
Effect of Priors on Reservoir Responses

	Sand Sill	Shale Sill	Sand Range	Shale Range
Low	2.25	0.25	250	250
Medium	16	1	500	500
High	36	4	1000	1000

- A simple two level (low and high) full factorial designs is chosen
  - Responses are different between 0.30 to 0.65 recovery factor
  - Response surface indicate Sand sill to be the dominating factor
  - Stochastic fluctuations are comparable to prior variations
- Six replicates at (16,1,500,500) and (36, 1, 500, 500)
  - Welch two-sample *t*-test indicate that the mean responses are different with $p = 0.09243$
  - Prior specifications has a statically significant effect on response

3D Problem : Honoring Seismic Trends

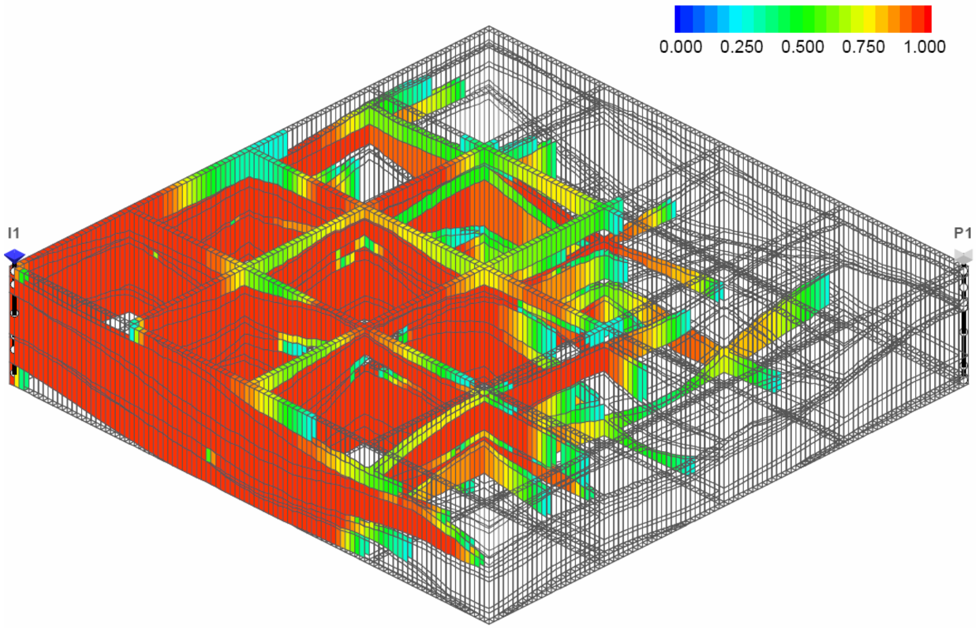


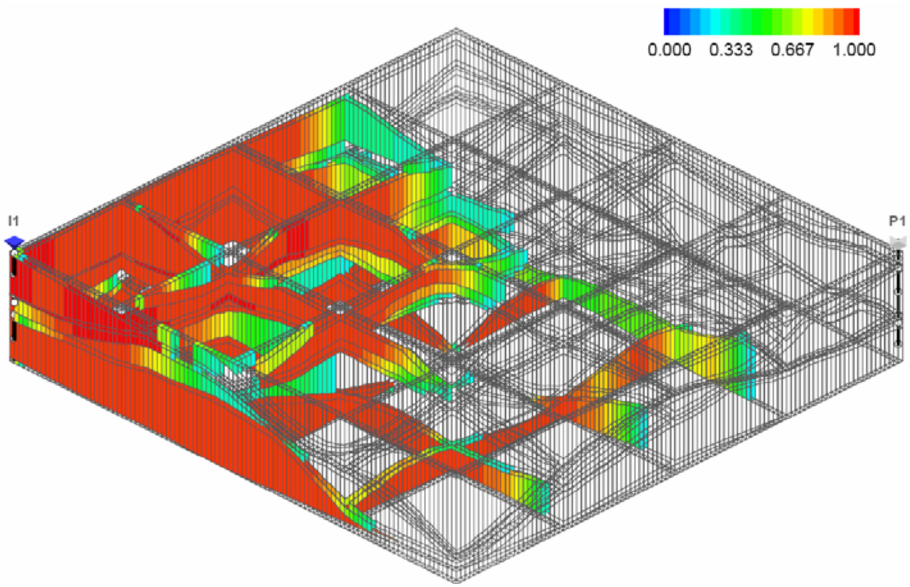


(a) Medium Sand Sill (4m²) **(b) High Sand Sill (36m²)**

- Simulations on a 100 x 100 x 10 cornerpoint grids with 4 conditioning data
- H = 20 m, H_s=14 m, $\Phi = 0.25$, and $\Phi H_s = 3.5$ m

3D Problem : Tracer Concentrations



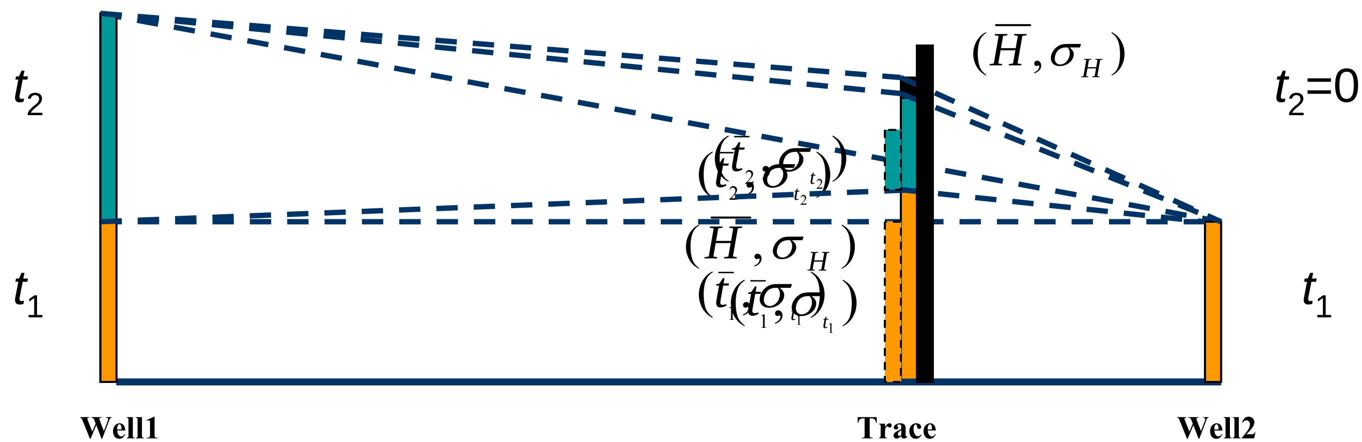


(a) Tracer in Medium Sill Case

(b) Tracer in High Sill Case

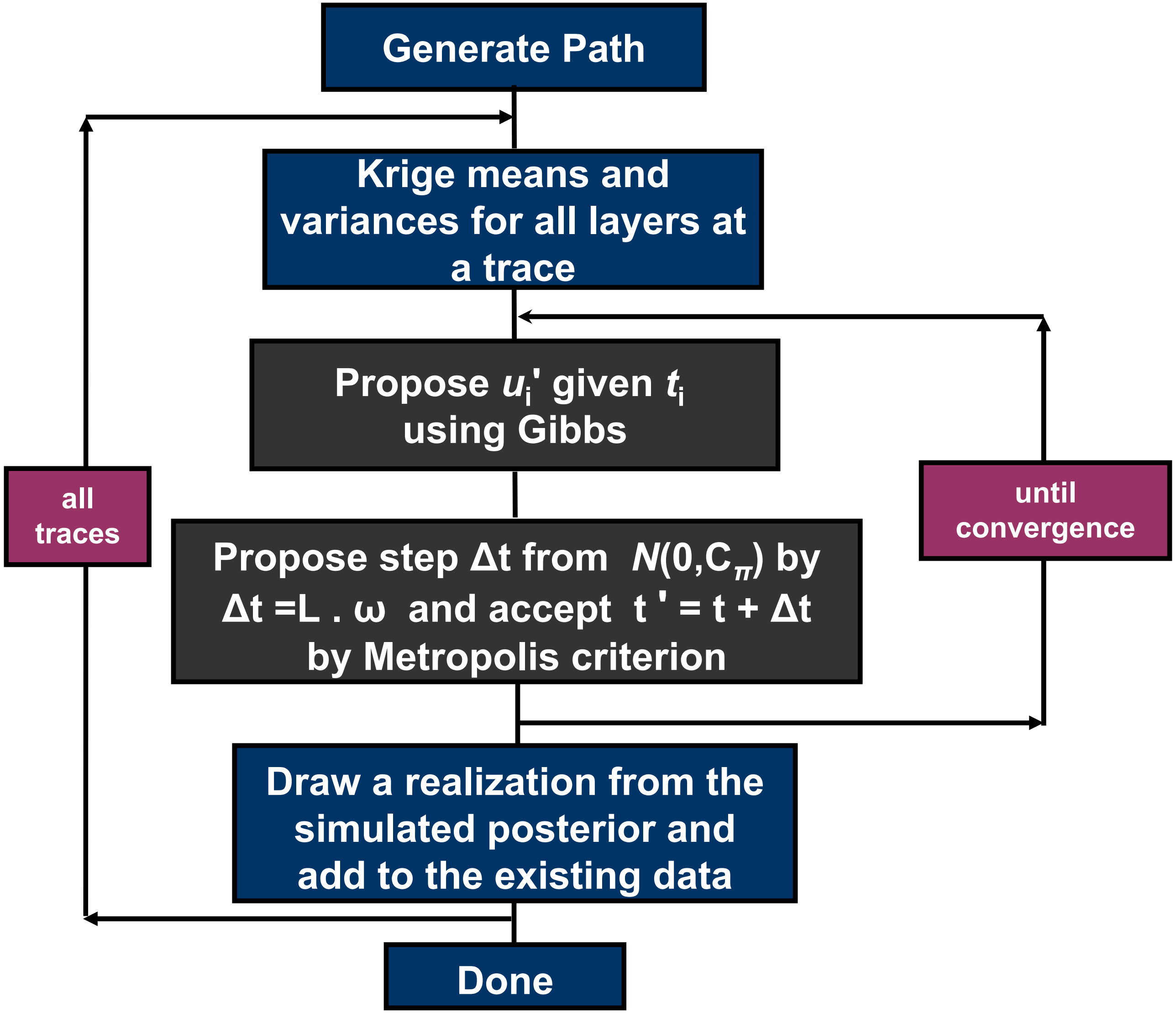
- Tracer concentrations before the break through
- Lower recovery in high sill case is caused by
  - Variability in thickness
  - More frequent terminations

A Simple Two layer Case

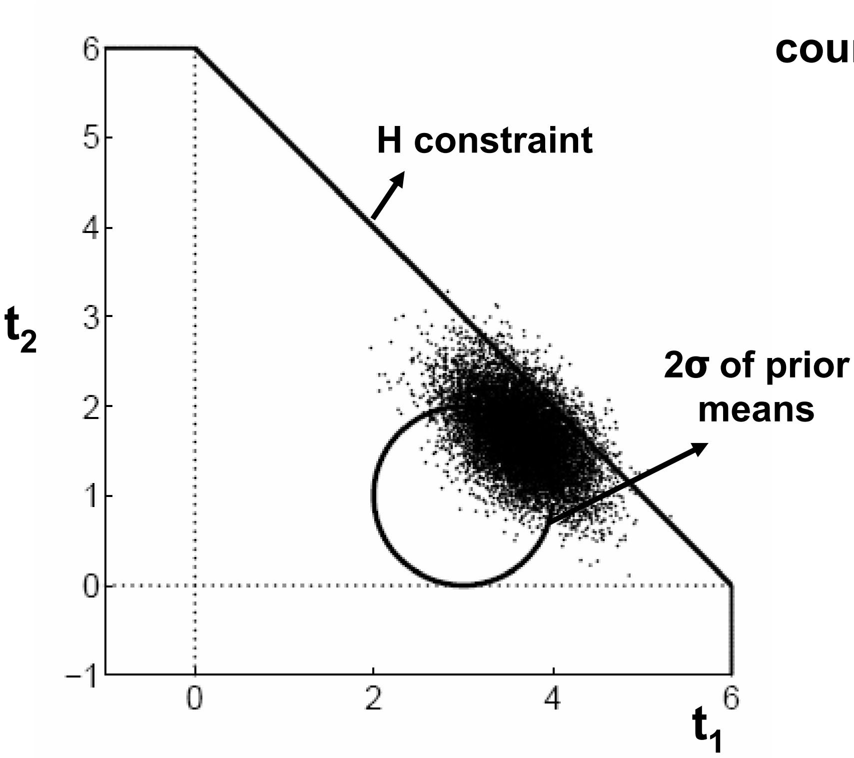


- Bayes reconciles seismic and well/continuity data
  - Posterior covariance weights each data type appropriately
- Simulation retrieves the complete distribution, not just the most likely combination

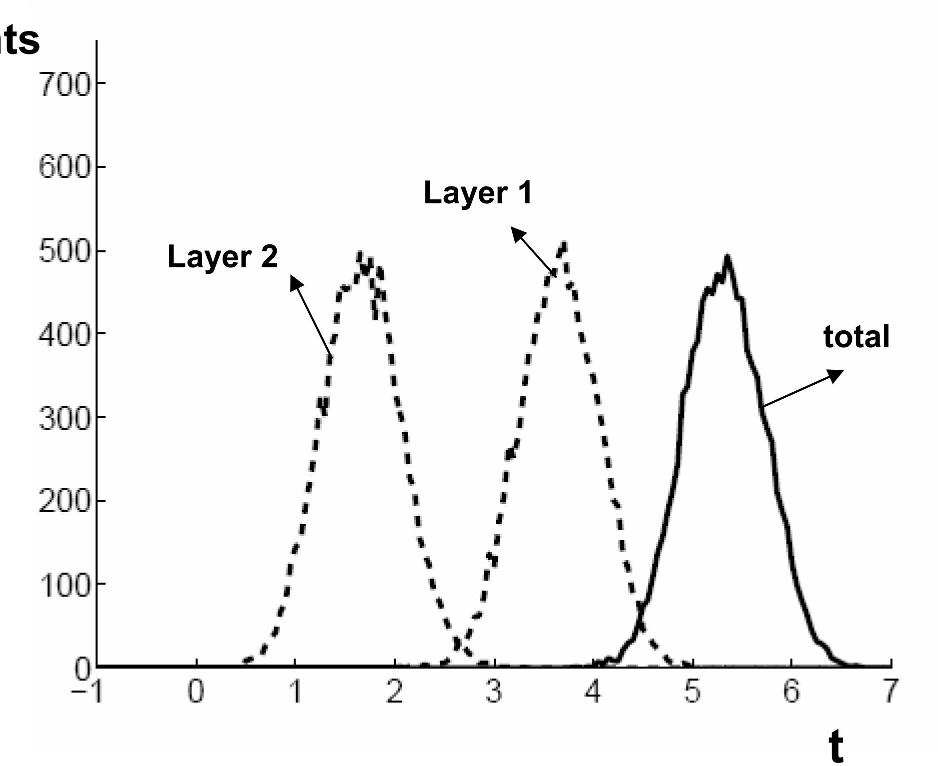
Sequential TG-MCMC



Prior not equal to weak constraint



Scattergram, N=8000



Histogram of t

$$\overline{H} = 6\text{ m},\ \sigma_H = 0.5\text{ m}$$

$$\bar{\mathbf{t}} = (3\text{ m}, 1\text{ m}),\ \sigma_t = 0.5\text{ m}$$

Performance Summary

Process	Work in Seconds
Kriging Work	1.7
5000 samples, all traces	310.5
Total cost of simulation	314.7
Using 2 GHz Pentium-M processor with 1 GB of RAM	
Implemented in ANSI C, g77 compiler, using NR & LAPACK routines	

- 5000 samples for 10^5 unknowns in 5min on a laptop
- 98% of computation is for generating and evaluating steps
- Fewer samples could be used in practice

Assumptions and Performance

- Layer thicknesses are vertically uncorrelated at each trace
- Lateral correlations are identical for all layers
- Toeplitz form for resolution matrix in non-exact constraint

$$\mathbf{G} = \begin{pmatrix} \frac{1}{\sigma_t^2} + \frac{1}{\sigma_H^2} & \frac{1}{\sigma_H^2} & \cdots & \frac{1}{\sigma_H^2} \\ \frac{1}{\sigma_H^2} & \frac{1}{\sigma_t^2} + \frac{1}{\sigma_H^2} & \cdots & \frac{1}{\sigma_H^2} \\ \vdots & \ddots & \ddots & \vdots \\ \frac{1}{\sigma_H^2} & \frac{1}{\sigma_H^2} & \cdots & \frac{1}{\sigma_t^2} + \frac{1}{\sigma_H^2} \end{pmatrix}$$

- Efficient Toeplitz solver
  - Handles layer drop-outs or drop-ins without refactoring

End of Slides