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A27 Downscaling Seismic Data to the Meter Scale – Sampling and Marginalization

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SUMMARY

Mesoscale (~ 10 m) models using rock-physics concepts and effective-media ideas are a manageable basis for Bayesian seismic integration because seismic is usefully informative at this scale. An attractive route to geocellular scale (~ 1 m) models is downscaling mesoscale models using categorical (facies) simulations that honor effective media laws, and using well data and geologic concepts to formulate priors.

In this nonlinear downscaling, it is unclear whether the overall posterior distributions for fine-scale models can be approximated as the product of conditional distributions using local neighborhoods, which is necessary for accurate sequential simulation. The factorization requires computing analytical marginal distributions (integrating over "unvisited" sites) and conditional distributions dependent only on "visited" sites. Analytical techniques fail for nonlinearly constrained problems; the only alternatives are expensive MCMC or analytical approximations within a sequential method. An approximation based on an expansion assuming weak correlation between visited and unvisited sites is developed in this paper.

We test and illustrate by comparing global methods (with rigorous marginals) to local approximations. Local method errors increase as correlation length increases, especially if seismic data are highly informative or the marginals are poorly approximated. Using the proposed marginal approximation improves sequential simulation accuracy for these cases.



1 Introduction

Beds thinner than $\sim 10 \text{ m} (\lambda/4; \lambda \text{ is the dominant wavelength})$ are poorly resolved in 3-D seismic data at depths greater than $\sim 3000 \text{ m}$. This limit and errors in seismic-derived property estimates complicate use of seismic data. However, these data can be used to infer external geometry and guide subseismic stratigraphic models. An attractive implementation of this idea is to incorporate seismic data in meso-scale Bayesian seismic inversions that treat subseismic heterogeneity *via* effective-media theory, and subsequently downscale these inversions to meter-scale models using constraint equations embodying the effective media laws (Gunning et al., 2007). In particular, downscaling specific realisations drawn from the posterior of the meso-scale inversion produces constraint equations for fine scale models.

The proposed approach models layer thicknesses as "marked–surfaces", with truncations of negative thicknesses to allow pinchouts. For example, a set of K sublayers (thicknesses $t_k, k \in \{1 \dots K\}$) of a meso-scale layer of total interval thickness H implies the downscaling constraint for each column

$$\sum_{k=1}^{K} \max(t_k, 0) = H$$

The constraint is embedded in a likelihood expression to yield a Bayesian posterior

$$\pi \left(\mathbf{t} | H, \mathbf{d} \right) \propto L \left(H | \mathbf{t}, \mathbf{d} \right) p \left(\mathbf{t} | \mathbf{d} \right)$$
(1)

where **t** is a vector of thicknesses, H is the "target thickness", and **d** any additional hard data. The posterior distribution for the fine scale parameters is in general high dimensional, so we seek a sequential simulation algorithm passing over all columns of the grid. Each column is simulated by sampling from a Bayesian posterior distribution conditional on hard data and previously visited columns *via* the priors, and collocated coarse scale constraints *via* the likelihood. A suitable likelihood, with "accuracy" σ_H for K layers at a column with expected total net-sand thickness H is

$$L(H|\mathbf{t}, \mathbf{d}) \propto \exp\left[-\left[\left(\sum_{k=1}^{K} \max(0, t_k)\right) - H\right]^2 / 2\sigma_H^2\right]$$
(2)

The prior distribution for the K layers is determined by kriging surrounding layer thickness (using data and previous simulations); the distributions are $\mathbf{t} \sim N(\bar{\mathbf{t}}, \mathbf{C}_p)$, where $\bar{\mathbf{t}}$ and \mathbf{C}_p are the kriged estimates and errors, respectively. A local, linearised posterior covariance derived from (1) is

$$\tilde{\mathbf{C}} = (\mathbf{C}_p^{-1} + \mathbf{X}\mathbf{X}^T / \sigma_H^2)^{-1}$$
(3)

where **X** is a design matrix comprising 1's if a layer k is present $(t_k > 0)$ and zero otherwise; **X** depends on **t**. This nonlinearity makes the posterior a piece-wise Gaussian, which is difficult to sample from.



2 Linear Theory

If we partition the model vector \mathbf{t} into I parts $(\mathbf{t}_1 \ \mathbf{t}_2 \ \dots \ \mathbf{t}_I)$ and the likelihood is a simple product over these parts, the posterior π can be written in the form

$$\pi(\mathbf{t}) = \pi(\mathbf{t}_1)\pi(\mathbf{t}_2|\mathbf{t}_1)\dots\pi(\mathbf{t}_I|\mathbf{t}_1\dots\mathbf{t}_{I-1})$$
(4)

which is suitable for sequential simulation. $\pi(\mathbf{t}_i|\mathbf{t}_1 \dots \mathbf{t}_{i-1})$ is the marginal distribution

$$\pi(\mathbf{t}_{i}|\mathbf{t}_{1}\dots\mathbf{t}_{i-1},\mathbf{d}) = \int_{-\infty}^{\infty} \prod_{j=i+1}^{I} L\left(\mathbf{H}_{j}|\mathbf{t}_{j},\mathbf{d}\right) p\left(\mathbf{t}|\mathbf{d}\right) \mathrm{d}\mathbf{t}_{i+1}\dots\mathrm{d}\mathbf{t}_{I}$$
(5)

Eqn. (5) is integrated over all $\mathbf{t}_j, j \in \{(i+1) \dots I\}$. Consider the basic partitioning into current (\mathbf{t}_1) and "unvisited" (\mathbf{t}_2) sites, $\mathbf{t} = (\mathbf{t}_1 \mathbf{t}_2)$. Assume the prior $\mathbf{t} \sim N(\boldsymbol{\mu}, \mathbf{C})$, observations $\mathbf{y} = (\mathbf{y}_1, \mathbf{y}_2)$ and error model $\mathbf{y} - \mathbf{X}\mathbf{t} \sim N(0, \mathbf{C}_y)$. Without loss of generality \mathbf{C}_y (usually diagonal) can be absorbed into \mathbf{X} and \mathbf{y} . Using the notation $\mathbf{\Sigma} = \mathbf{C}^{-1}$, the marginal for \mathbf{t}_1 given \mathbf{y} is then Normal, with covariance

$$\tilde{\mathbf{C}}_{11} = (\boldsymbol{\Sigma}_{11} + \mathbf{X}_1^{\mathrm{T}} \mathbf{X}_1 - \boldsymbol{\Sigma}_{12} (\boldsymbol{\Sigma}_{22} + \mathbf{X}_2^{\mathrm{T}} \mathbf{X}_2)^{-1} \boldsymbol{\Sigma}_{21})^{-1}$$
(6)

and mean

$$\tilde{\mathbf{t}}_1 = \tilde{\mathbf{C}}_{11} \mathbf{X}_1^{\mathrm{T}} (\mathbf{y}_1 - \mathbf{X}_1 \boldsymbol{\mu}_1) + \tilde{\mathbf{C}}_{11} \mathbf{X}_2^{\mathrm{T}} (\mathbf{y}_2 - \mathbf{X}_2 \boldsymbol{\mu}_2) + \boldsymbol{\mu}_1$$
(7)

In sequential simulation, the dimensionality of \mathbf{t}_2 (and rank of \mathbf{C}_{22}) can be very large. We need to approximate equations (6,7). One plausible approximation, based on a "weak correlations" expansion (small \mathbf{C}_{12}), yields

$$\tilde{\mathbf{C}}_{11} = (\mathbf{C}_{11}^{-1} + \mathbf{X}_{1}^{\mathrm{T}}\mathbf{X}_{1} + \mathbf{C}_{11}^{-1}\mathbf{C}_{21}^{\mathrm{T}}\mathbf{X}_{2}^{\mathrm{T}}\underbrace{\mathbf{X}_{2}\mathbf{C}_{21}\mathbf{C}_{11}^{-1}}_{\mathbf{X}_{2,\mathrm{eff}}})^{-1}$$
(8)

and

$$\tilde{\mathbf{t}}_1 = \tilde{\mathbf{C}}_{11}(\mathbf{X}_1^{\mathrm{T}}(\mathbf{y}_1 - \mathbf{X}_1\boldsymbol{\mu}_1) + \mathbf{X}_{2,\mathrm{eff}}^{\mathrm{T}}(\mathbf{y}_2 - \mathbf{X}_2\boldsymbol{\mu}_2)) + \boldsymbol{\mu}_1.$$
(9)

This removes the need to invert the (potentially very large) C_{22} matrix block. Equation (9) is a standard Bayesian formula to update \mathbf{t}_1 given \mathbf{y}_1 , with the contribution of secondary data \mathbf{y}_2 attenuated by the modified sensitivity matrix $\mathbf{X}_{2,\text{eff}}$. This is a manageable approximation for the marginal that includes the effect of information at unvisited sites.

3 Discussion and Examples

The above equations are for linear constraints $f(\mathbf{t}) = \mathbf{X}\mathbf{t} + \mathbf{t}_0$. For the nonlinear constraints $\mathbf{X} = \mathbf{X}(\mathbf{t})$ in the downscaling problem, additional approximations are needed to make the marginal for \mathbf{t}_1 tractable. If we neglect the nonlinearities in \mathbf{t}_2 then the marginal is analytically integrable:

$$\pi \left(\mathbf{t}_{1} | \mathbf{H} \right) \propto e^{-\frac{\left(f(\mathbf{t}_{1}) - \mathbf{H}_{1} \right)^{2}}{2\sigma_{\mathbf{H}_{1}}^{2}}} \int_{-\infty}^{\infty} e^{-\frac{1}{2} \left(\mathbf{X}_{2} \mathbf{t}_{2} - \mathbf{H}_{2} \right)^{T} \mathbf{C}_{\mathbf{H}_{2}}^{-1} \left(\mathbf{X} \mathbf{t}_{2} - \mathbf{H}_{2} \right)} p\left(\mathbf{t}_{1}, \mathbf{t}_{2} \right) d\mathbf{t}_{2}$$
(10)



which we call sequential simulation with marginalization (SM). A heavier approximation neglects lateral correlations between the current and unsimulated columns, t_1 and t_2 , yielding sequential simulation without marginalization (SS),

$$\pi\left(\mathbf{t}_{1}|\mathbf{H}\right) \propto e^{-\frac{\left(f(\mathbf{t}_{1})-\mathbf{H}_{1}\right)^{2}}{2\sigma_{\mathbf{H}_{1}}^{2}}}p\left(\mathbf{t}_{1}\right)$$
(11)

A small 10 layer \times 10 column 2D example compares SM and SS algorithms with a global MCMC method (GM). Columns have 100 m separation. Two constraining cases of constraint uncertainty (σ_H) and lateral correlation (range, λ_x) are considered. The seismic thickness constraint (H = 20 m) and σ_H are stationary. The prior means (for μ_1 in Eqn. (9)) are $\bar{t}_k = 2$ m $\forall k$; autocovariances are Gaussian with sill related to a stationary prior standard deviation $\sigma_t = \bar{t}_k$. These parameters cause a high probability for layers to pinch out, which is a feature of particular interest.



Figure 1: Global (GM), sequential marginalized (SM), and standard sequential (SS) simulation results for various cases. Fractions of zero-thickness beds are given in the legend. Results are for layer 1.

Weak seismic constraint, strong geologic correlation ($\sigma_H = 5$ m and $\lambda_x = 2000$ m). The marginals for the first column visited for the global, marginalized, and standard sequential methods [Fig.(1a)] have only small differences between them. Similar characteristics are observed if geologic correlations are weaker.

Strong seismic constraint and strong geologic correlation ($\sigma_H = 0.5$ m and $\lambda_x = 2000$ m). The marginals for the global method are narrower than the standard sequential method. The approximate marginal method is closer to the rigorous MCMC result. For tighter constraints, auxiliary variables are required for sampling (Kalla et al., 2006).

4 Conclusions

Sampling the uncertainty in these nonlinear downscaling problems is difficult. Global MCMC methods are accurate but expensive, which motivates consideration of sequential



methods. Cheaper sequential methods are reasonably accurate if the lateral correlation is not high, and the constraints are weak. If the correlation is high and constraints are strong, naïve sequential simulation poorly approximates the marginals. For such cases, the proposed approximate marginals offer improved sampling at moderate cost.

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