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Bayesian Approaches to Resolution in CSEM

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SUMMARY

Inversion of CSEM data is a challenging problem for at least two reasons. (1) The inverse problem is significantly nonlinear in the subsurface resistivity parameters, and (2) very poor scaling results from the strong absorption of the incident EM energy. We show not only that a Bayesian approach furnishes both the extra information required to stabilise the inverse problem (a well-known fact), but also the little-known fact that it provides a coherent method for estimating inferable resolution. This is demonstrated for single models with spatial correlations, and also for model comparison within families of models with variable spatial discretization. The Bayesian approach is thus the necessary key to understanding uncertainty within models, but also uncertainty between models, model resolution, and multidisciplinary data integration.

Introduction

In recent years, the controlled source electromagnetic (CSEM) technique has become a popular tool in the hydrocarbon exploration game. The method is naturally suited to detecting resistive anomalies in the marine subsurface, hopefully due to the presence of hydrocarbon deposits. Sufficient geological knowledge must be available to exclude the possibility of highly resistive rocks like evaporites, volcanics, or carbonates. Taken in conjunction with seismic data for geological and structural delineation, the tool is potentially a powerful discriminator between high and low gas saturation. The technique is a valuable complement to seismic methods, since these are well known to have difficulty in detection of gas saturation in AVO applications.

Many articles have appeared to date outlining the general nature of the CSEM acquisition framework (see, e.g. the CSEM "special section" of Geophysics, vol 72, No. 2). In the domain of applicability of the method, two main issues limit the usefulness of the technique. Firstly, the heavily dispersive nature of the energy propagation destroys all bandwidth above a few hertz, which automatically imposes severe restrictions on resolution. Secondly, the dynamic range of conductivity from seawater to resistive anomalies (or deeper rocks), is usually at least several decades, so, in an inverse-problem context, the subsurface response $F(m)$ is very poorly modelled as a "small" deviation from some "agnostic" reference model m_0. In other words, the Born approximation, so central to seismic imaging, is rarely very useful for real CSEM data. This strong nonlinearity means the optimization surface is nearly always multimodal, badly scaled, and contorted in shape.

These issues make meaningful 2 or 3 dimensional CSEM inversion a particularly difficult problem. Since the problem is very poorly scaled, inverse approaches require additional terms to stabilise or better-condition the iterations. For diverse reasons, the bulk of the inverse-theoretical work done in the EM community is not fundamentally statistical in nature, but rather approaches the stability problem using pragmatic Tikhonov-regularization methods. This introduces the awkward problem of how to estimate and justify the free parameters in these regularizing operators, and make meaningful statements about what these pieces imply about model resolution and uncertainty.

These conceptual difficulties disappear if a Bayesian approach to the inverse problem is taken. Modern Bayesian inference frameworks allow inverse problems to be stated in terms of an inference problem for the posterior distribution of a suite of model parameters and possible meta-parameters, and questions about resolution or uncertainty are answerable directly from this posterior distribution. Further, since such statements are conditional on the chosen model, a framework that enables sensible comparison of different models, or families of models - even of varying dimensionality - is very useful. Such an apparatus is available in approaches variously called Bayesian Model Selection, or Bayesian Mixture Modelling (Hoeting et al (1999)). A further argument for Bayesian approaches is that they are easily the most natural way to introduce knowledge from other data sources or professional expertise, with its requisite precision and interdependencies etc, via additional likelihood terms or priors. This is important for CSEM, since CSEM data is unlikely to be used in isolation for a major decision.

We illustrate here how two Bayesian approaches to the resolution problem in CSEM inversion yield parsimonious, optimally resolved models for the test case of 1D CSEM data. The theory here provides a more formal and satisfactory account of how to infer resolution than the "discrepancy principle" (Farquharson and Oldenburg, 2004) of the well-known OCCAM code (Constable *et al*, 1987). Extensions to higher dimensions will be natural.

Theory

Resolution is most effectively understood as an interaction between the particular spatial resolution of a inversion model, and the effective number of degrees of freedom which can be meaningfully estimated from the data. There are two distinct approaches to resolution inference:

- Firstly, if a particular "fine" spatial model m is supplemented by well chosen meta-parameters μ expressing spatial correlation, the resolution is embodied in the way the metaparameters control the local eigenvalue or singular value structure near the most-likely points in the parameter space. Overfitted, or excessively deconvolved, models correspond to a low-probability region of the meta-parameter posterior distribution. We embed the meta-parameters μ in a multi-Gaussian correlated spatial prior $N(m_p, C_p(\mu))$ for log(resistivity). Thus, with data d, a forward model $F(m)$,and Gaussian noise $N(0, C_d)$, the joint Bayesian posterior (likelihood $L(d|m)$ $\times$ prior $p(m, \mu)$) is maximised at the optimum of

  $$2\chi^2_{\text{Bayes}} = (d - F(m))^T C_d^{-1}(d - F(m)) + (m - m_p)^T C_p(\mu)^{-1}(m - m_p) + \log|C_p(\mu)|$$

  The balance between the smoothing (second) term and its $\log|C_p(\mu)|$ normalisation determines the inversion resolution. This occurs naturally and without need to invoke "target" values for the data-misfit contribution (first term). A related idea is to estimate μ by the maximum of its marginal distribution ($\pi(\mu) \sim \int L(d|m)p(m, \mu)dm$), sometimes called the Empirical Bayes estimate (Mitsuhata, 2004).

- Resolution can be inferred by performing model-selection over a family of models with variable spatial discretization. For ranking of model probabilities, we use the marginal model likelihood (MML) $\pi(k)$, obtained by integrating the Bayesian posterior density over the model parameters m_k;

  $$\pi(k) = \int L(d|m_k)p(m_k)dm_k.$$

  This is sometimes called the *evidence* (Sambridge et al, 2006). In general, the integral is quite difficult to perform, but approximations like the Laplace approximation are very effective if the posterior is modestly compact (this requires, essentially, the determinant of the Hessian at the global optimum). It is known that the Laplace approximation behaves asymptotically like the celebrated Bayes Information Criterion (BIC) (Denison, 2002) and thus the MML, like the BIC, has the required "Occamist" characteristic of favouring the simplest model that adequately explains the data. We perform model comparison by recursively splitting a very coarse vertical 2–layer model, retaining splits where they lead to an improvement in the MML. This surprisingly simple idea yields solutions with resolution where the sensitivity is greatest, and typically suppresses detail in the deeper subsurface.

For both methods, a globalizing optimization strategy is important. We usually fire off a suite of local optimizers, starting at suitably dispersed starting points. Some known symmetries can be used to spread the starting points, but randomisation is also usually effective. Using only the dominant optima is a respectable approximation for the integrals needed. Uncertainties in the inferred parameters can be estimated from the covariances arising from local linearization at modes, but usually these are too small: the dispersion between distinct solutions is usually much greater, and that between alternative models may be greater still. Full assessment of uncertainties is in-principle possible using MCMC methods, but these must be specially tailored with proposal kernels suited to the multimodal posterior and poor scaling. It is important to remember that the full posterior distribution embodies what can be claimed or inferred from geophysical inversion, not just particular point estimates or moments of the posterior.

Example

A typical 1D CMP inversion using both meta-parameters and model-splitting is shown in Figures 1 and 2. Electric field amplitude CSEM data only is used to invert for structure, using three frequencies (0.25, 0.75,1.25 Hz).

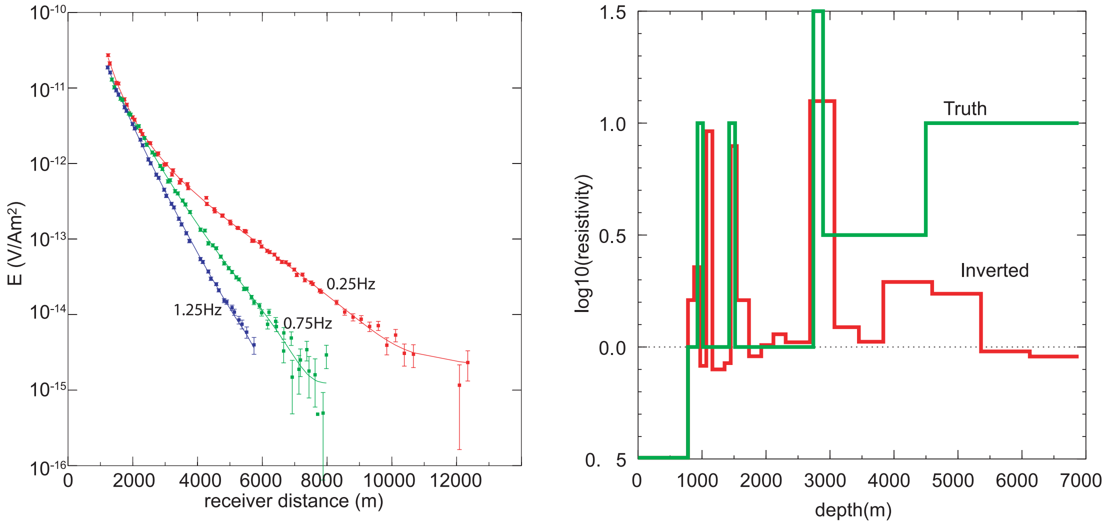

Figure 1: Left; Typical 3-frequency CMP amplitude data used for CSEM inversion, with fitted responses. Right: inversion using meta-parameters for smoothing a finer grid (red curve), compared to green curve truth case.

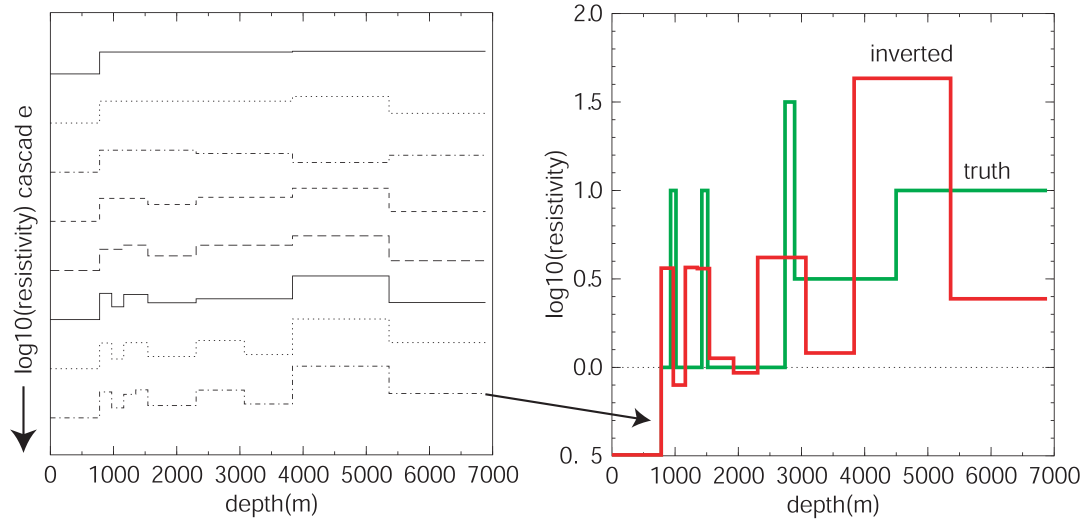

Figure 2: Left; Bayesian model-selection cascade of models used to determine vertical resolution. Algorithm terminates at the lowest model. Right: inverted model (red) against known truth case (green).

Both methods fit the data to the noise level satisfactorily. It is evident that neither of these methods is able to justify detailed structure at much depth, and the hierarchical method will clearly favour relatively coarse structures. For 1D models, even hundreds of data points (including phase, more frequencies etc) are unlikely to justify more than 10–20 inferable degrees of freedom.

The overhead is using these methods is very little compared to established methods. As usual, the bulk of the work lies in computing forward responses and sensitivity (or Frechet) matrices. The details above are easily incorporated into conventional Gauss–Newton schemes, with the relevant determinants etc easily computed from the Hessian matrices available at the termination of the optimization passes. Some bookkeeping is required for the global optimizations and the splitting schemes. The splitting schemes in particular can be rapid, since the models are very low dimensional.

Conclusions

Multimodality and poor scaling are pervasive problems in CSEM inversion. Bayesian approaches provide a consistent framework for addressing resolution issues in CSEM problems, and partially address the scaling problem. Optimal resolution can be extracted through either model selection methods or continuous meta-parameters controlling resolution within a single model. The meta-parameter spatial smoothing approach provides a rigorous basis for so–called "Occamist" inversion, showing that parsimony is a natural consequence of a Bayesian formalism. Multimodality is expressed through multiple peaks in the posterior probability surface, and this can only be address through combined globalization and enumeration strategies. Typical 1D CSEM data inverted in common–midpoint style suggest the data justifies only O(10) parameters per midpoint.

Acknowledgements

Funding support from BHPBilliton is gratefully acknowledged.

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