"Three Sisters" -- aboriginal womans' place for doing business, near BHPB Yandi iron ore mine

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Metropolis methods applied to Bayesian geologic inversions

Michael Glinsky (CEO Science Leader, CSIRO; Adjunct Professor of Physics, UWA)

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Outline

- Bayesian inversion with uncertainty as a statistical mechanics problem
- Application to seismic inversion
- Application to marine E&M inversion
- The future

The connection of Bayesian inversion to the Metropolis method of statistical mechanics

P(A^B) = P(A) P(B|A) = P(B) P(A|B)

Bayes' Theorem -- a probability commutation relation

A = m = model
B = d = observed data

know forward model

P(m|d) = P(m) P(d|m) / P(d)

want to know

prior probability of model

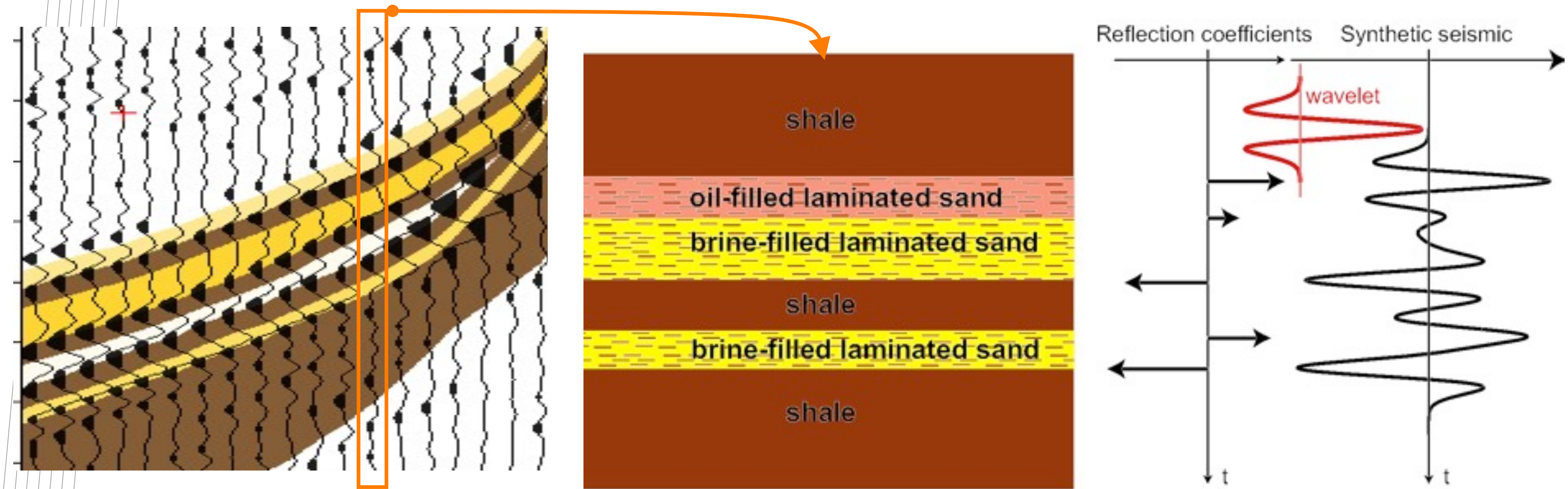
log[P(d|m)] ~ - [d - s(m)]^2 / noise^2 ~ - H / k_BT

$$\langle m \rangle = \int m P(m \mid d)\, dm = \sum_{i=1}^{N} \frac{1}{N} m_i \quad \text{for } \{m_i\}_{P(m|d)}$$

Probabilistic model based inversion

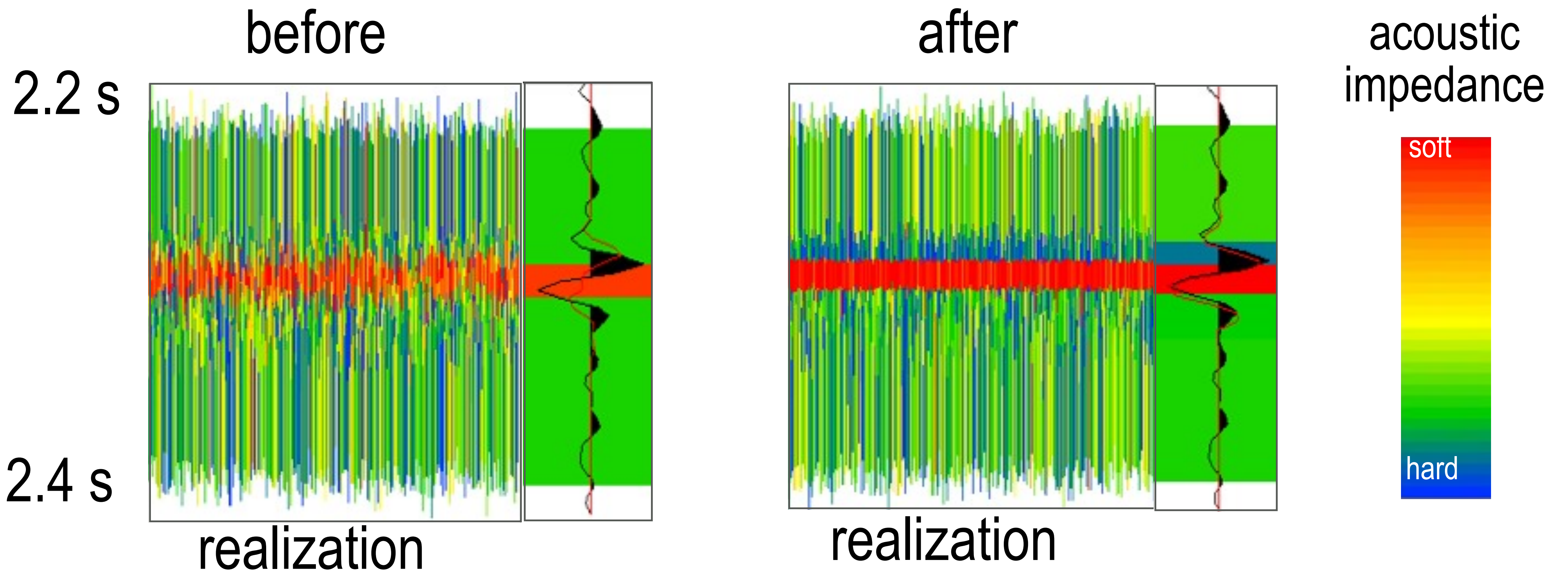
- Layer based model built at seismic loop scale using sparse spike inversion
- Standard rock physics correlations estimated with uncertainty
- Fundamental properties of layers are:
  - net-to-gross ratio (N/G)
  - layer top and base
  - fluid type
- Ensemble of models generated that are consistent with seismic to within estimated noise level

A closer look at Bayesian seismic inversion

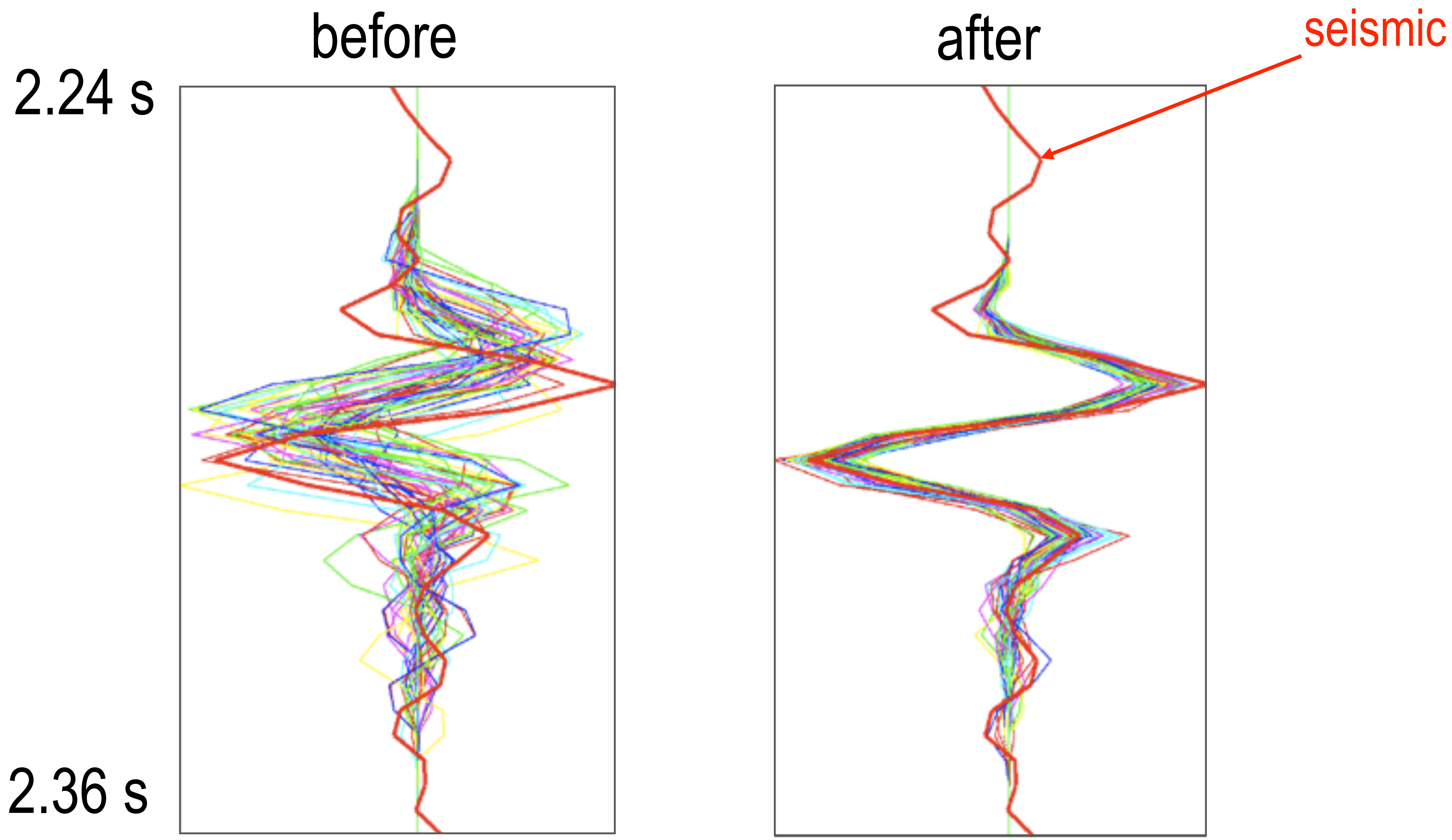


- Fundamental parameters
  - Layer times
  - Rock properties in each layer
  - Fluid type
- Forward model
  - Reuss/Gassman for fluids
  - Convolution (multi-stack)
- Priors
  - Regional rock trends, layer “picks”
- Likelihoods
  - Synthetic seismic
  - Isopachs
- Posteriors
  - Multimodel MCMC sampling

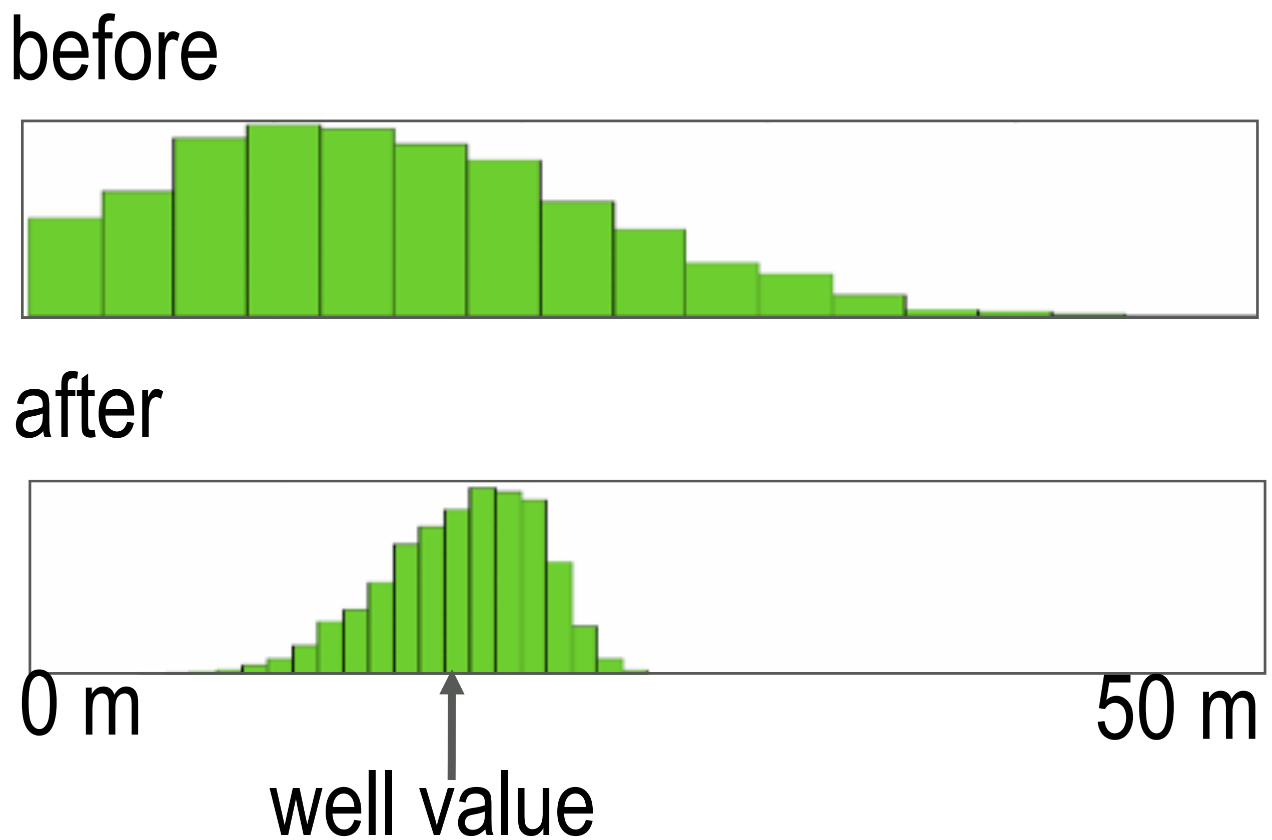
Ensemble of models at well location show effect of model based inversion



Effect of model based inversion on match of synthetic seismic to seismic data

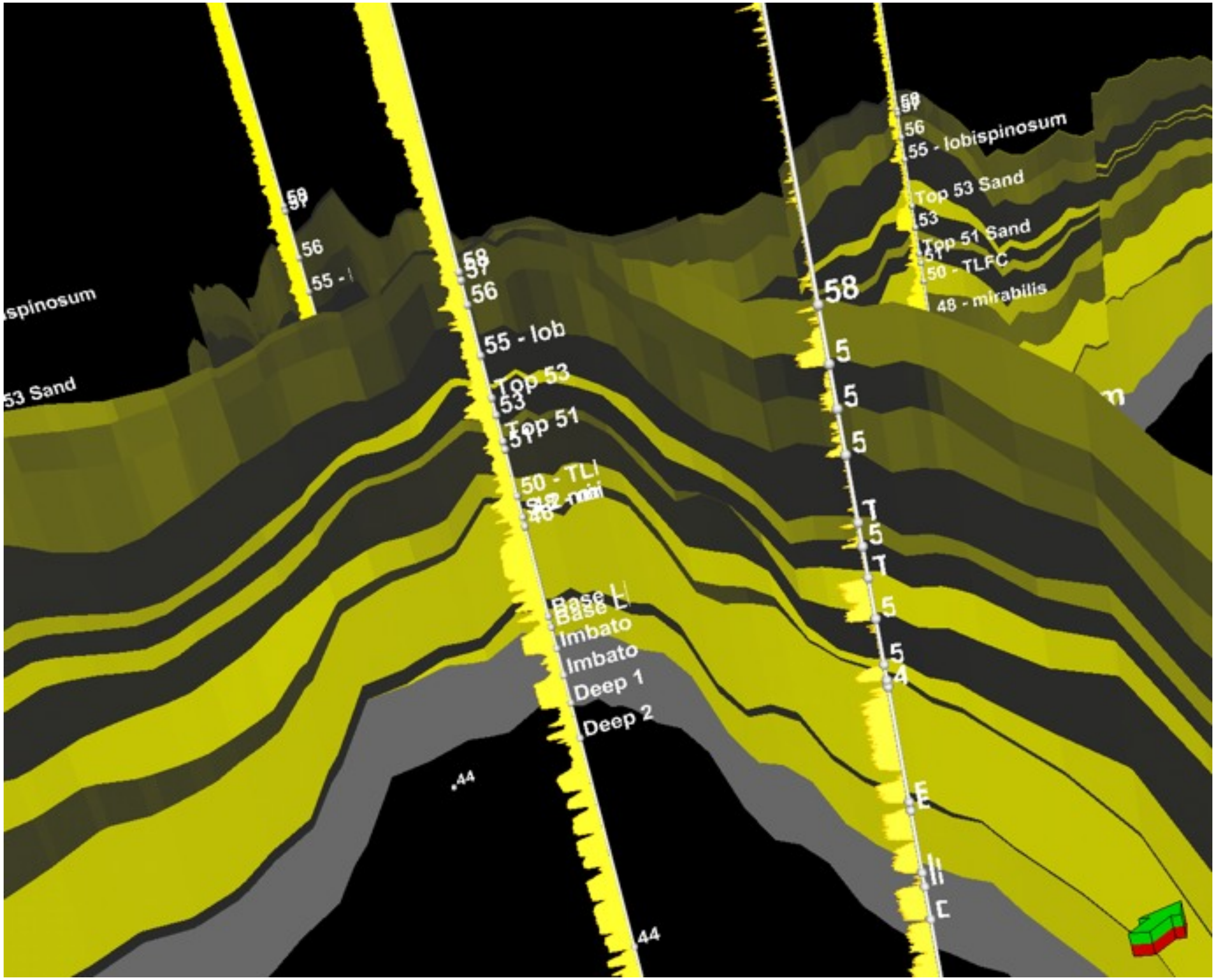


Inversion tightens the range of possible net sand

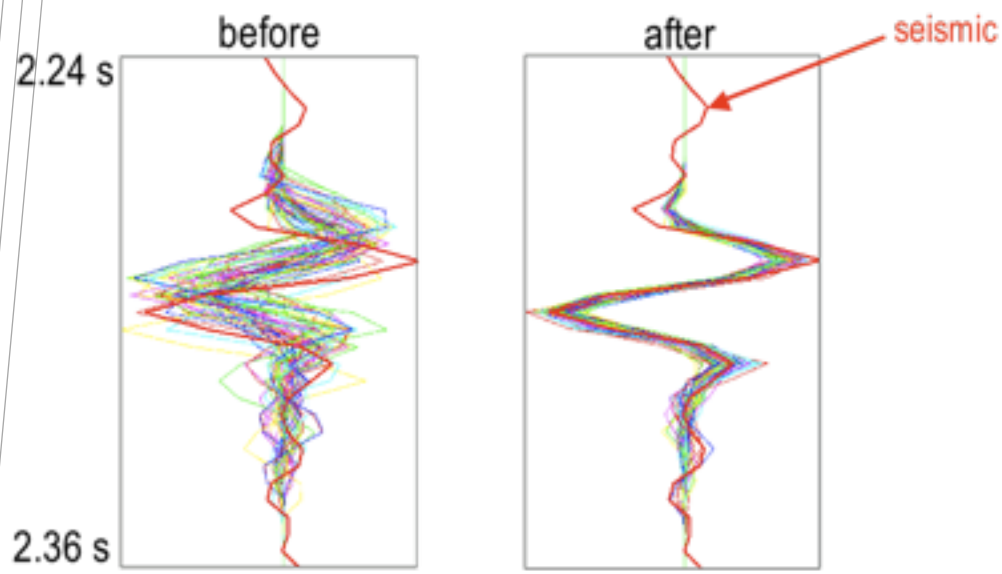


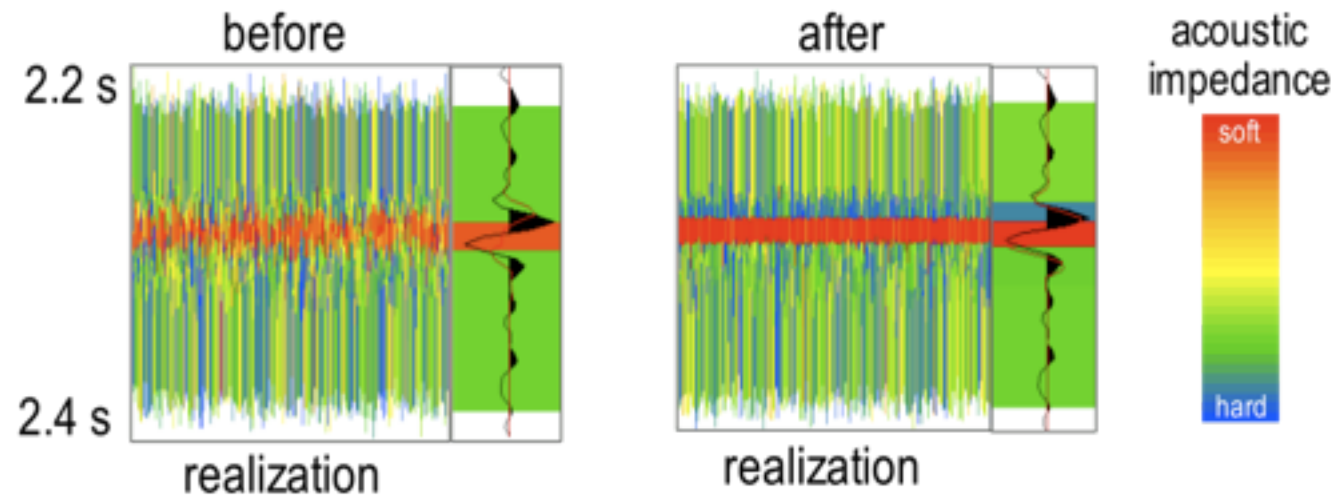
probability of oil increased to 97% from 50% (oil in sand at this location)

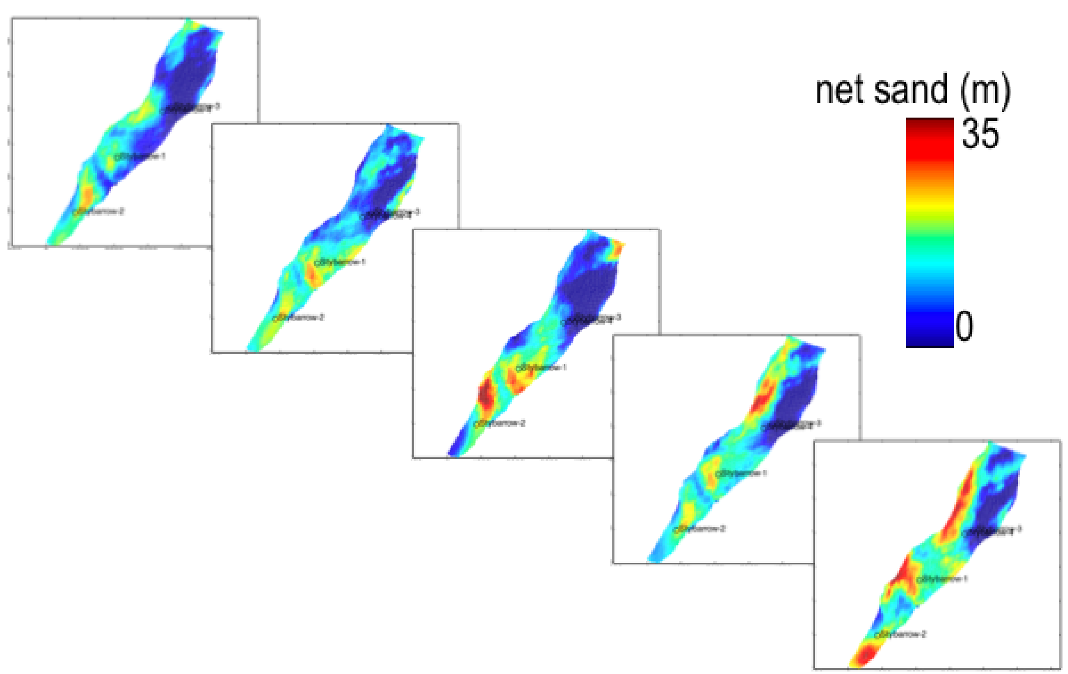
Imbed the result into 3D model

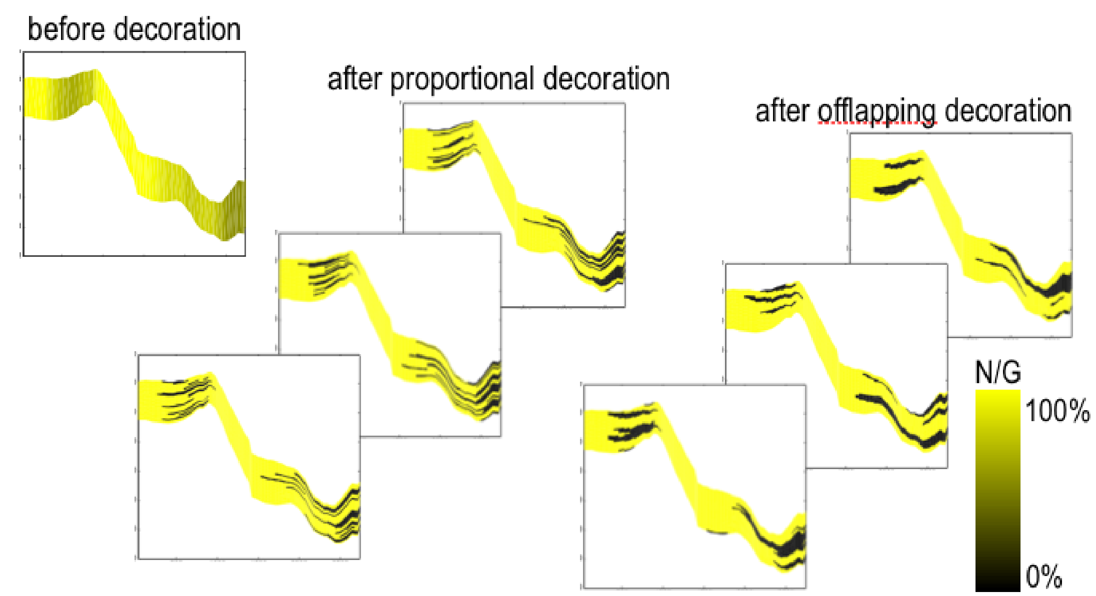


An overview of the model based inversion process









Another application of Metropolis method -- marine CSEM inversion

- description of the physics
- what makes the inversion difficult
  - tight & non-linear (parametric bootstrap & path finding)
  - multi modal (model selection)
  - bound constraints (projected newton methods)
- connection of Bayesian to classical methods
- flavors of Bayes
  - Bayesian smoothing
  - layer split/merge
  - log grid with no-smoothing
- benchmarks
  - wedge
  - “bird” model
- anomaly definition
- systematic noise

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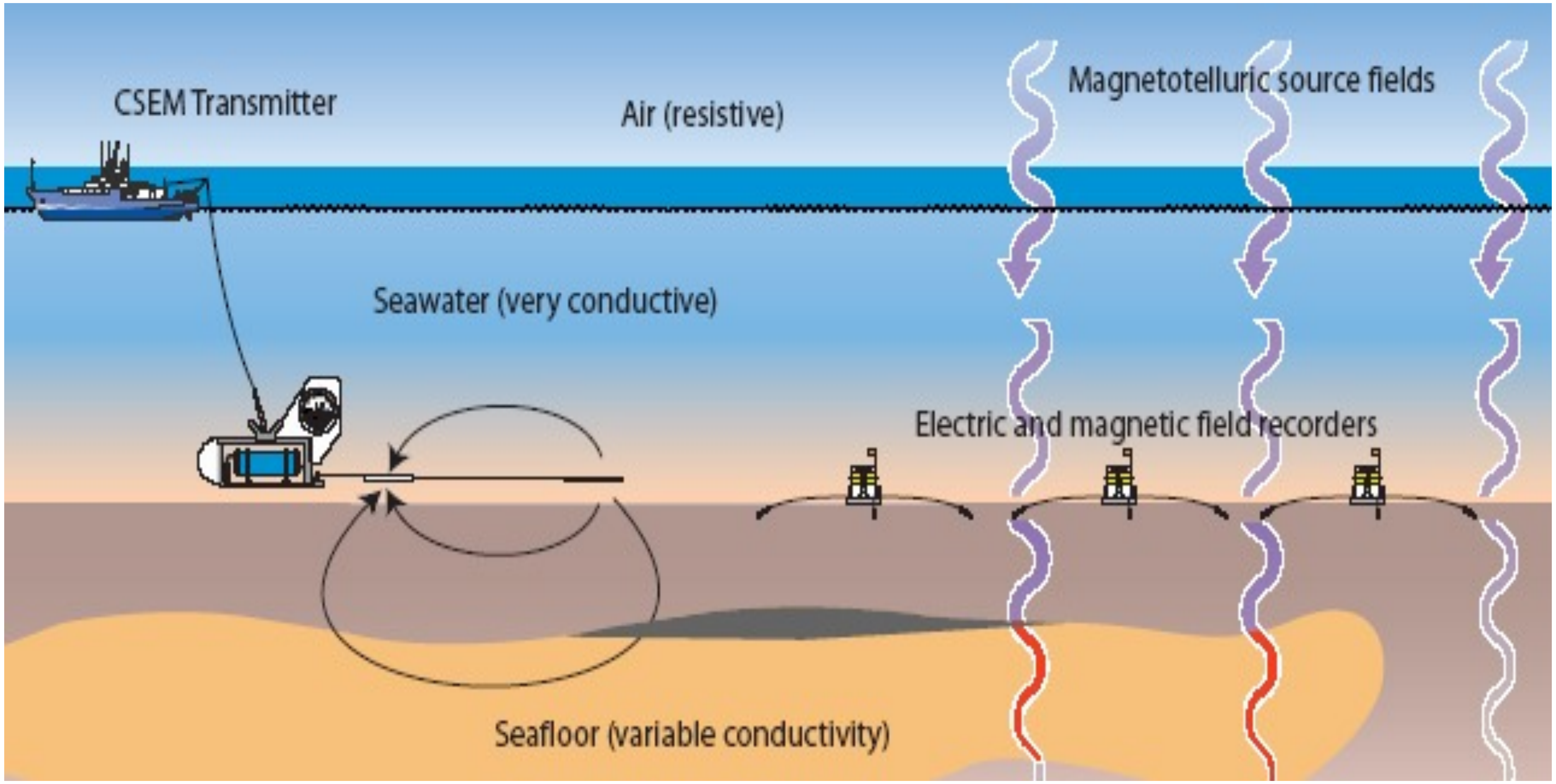
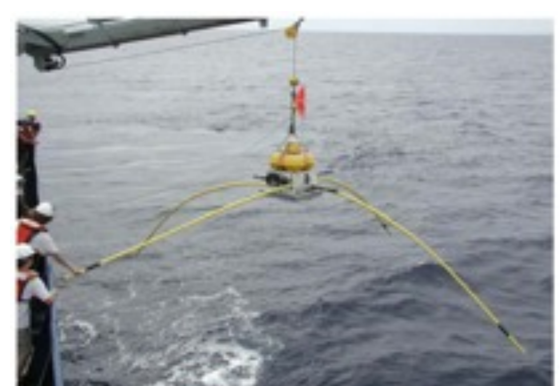
Controlled Source ElectroMagnetics (CSEM) or seabed logging (SBL)



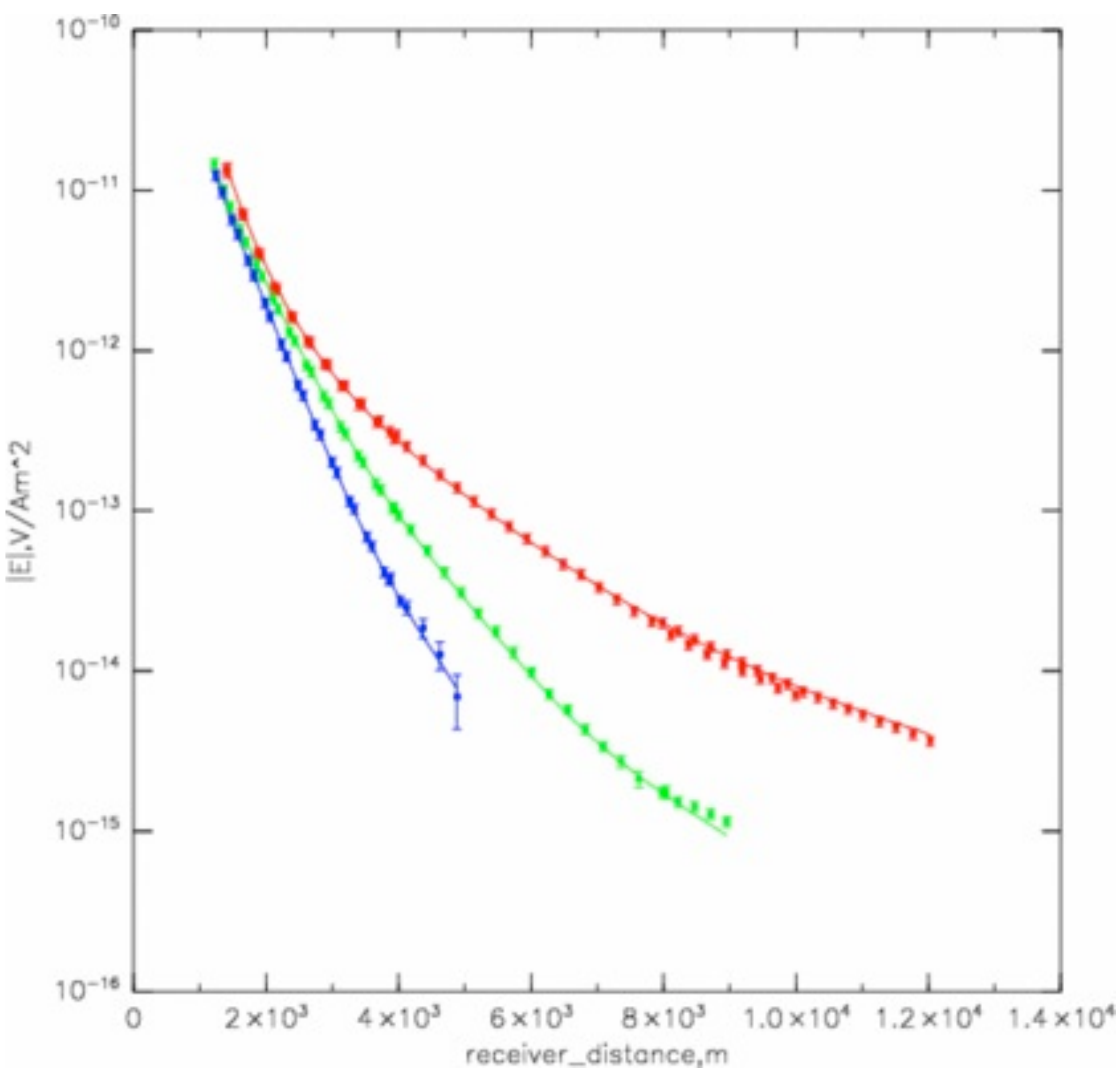
Appeal? A "hydrocarbon saturation" "reservoir quality" detector.

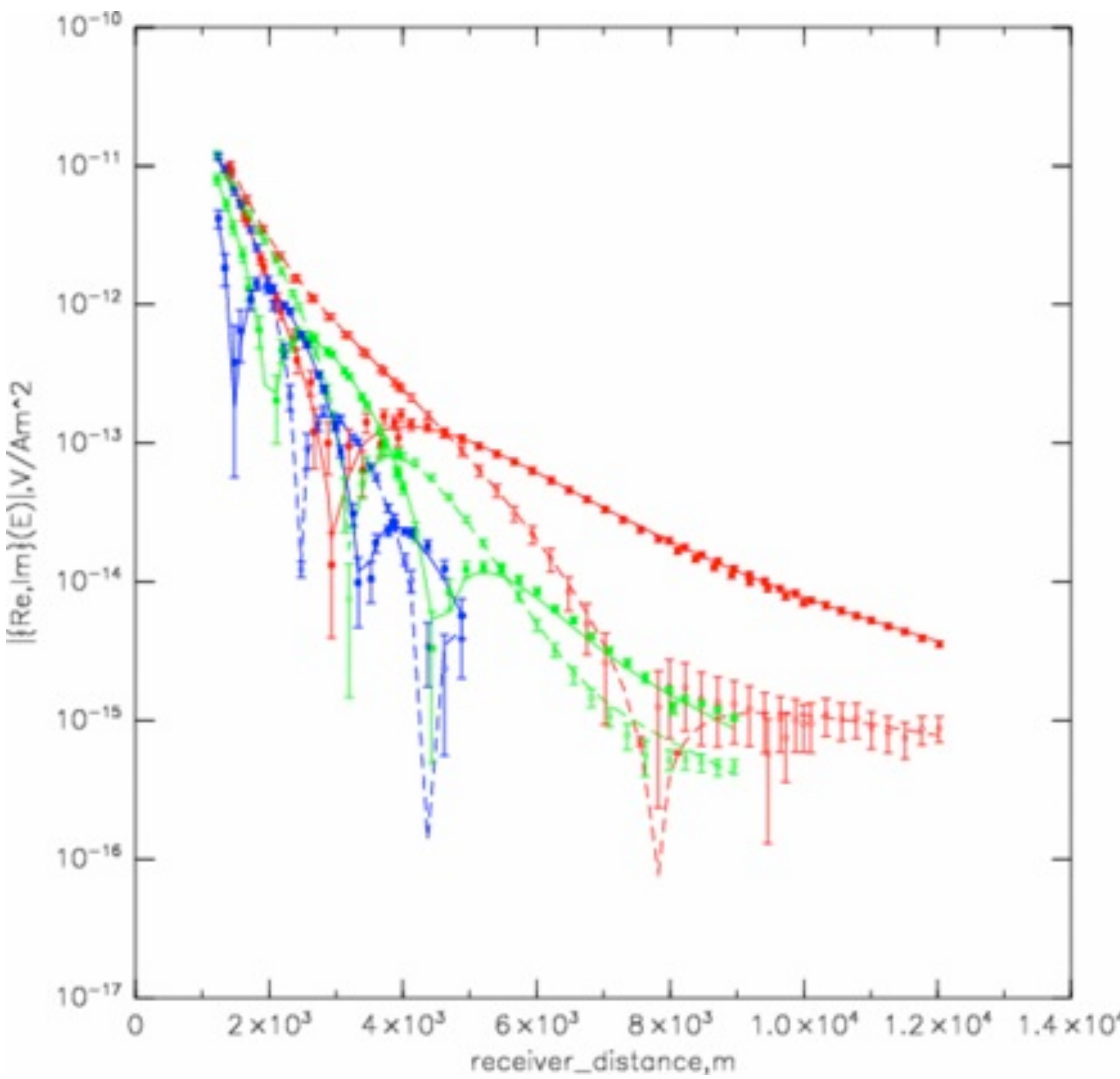
Very rapid growth of service companies since 2000. Psuedo-"bust" 2007-8.

Abundant data. Little consensus on interpretation.



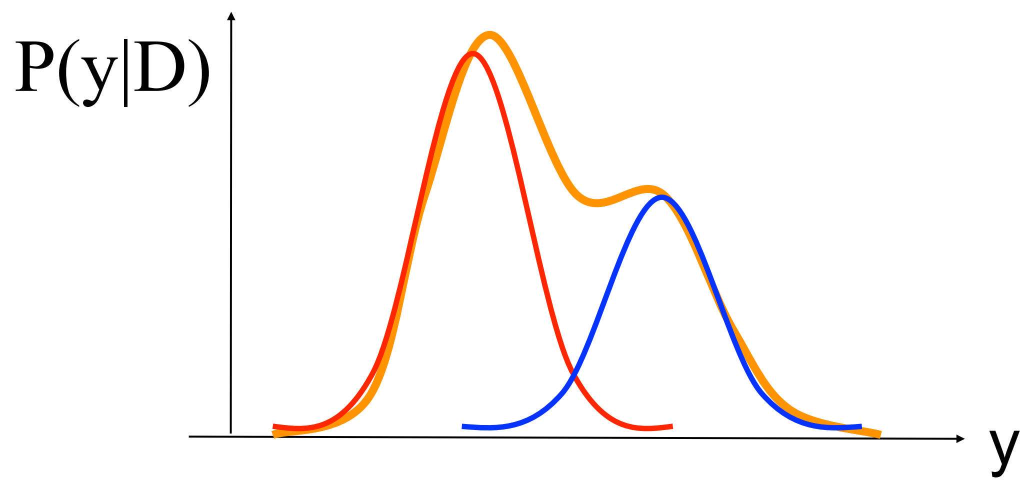
Typical data and a 1D layered earth model fit





Multi-modal distributions -- an inversion difficulty

- need to Bayesian model average for y
  - $P(y|D) = \sum_k P(y|M_k,D)\, P(D|k)$

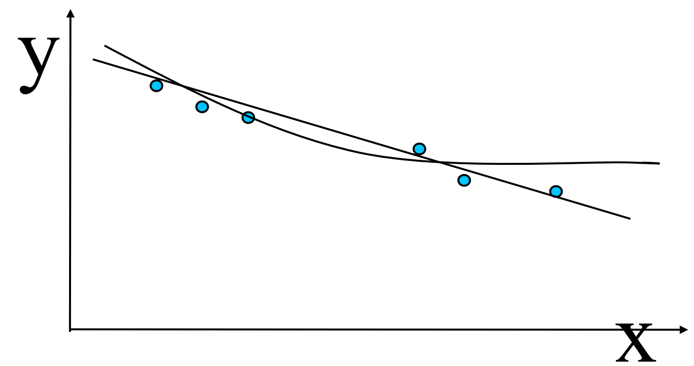


- because of mixture distributions
  - model geometry (e.g., number of layers)
  - rock types
  - fluids
- important because many times
  - uncertainties *within* a model < uncertainty between models

Model selection

Model selection – classic statistics problem

line ??, parabola ??, sextic ??



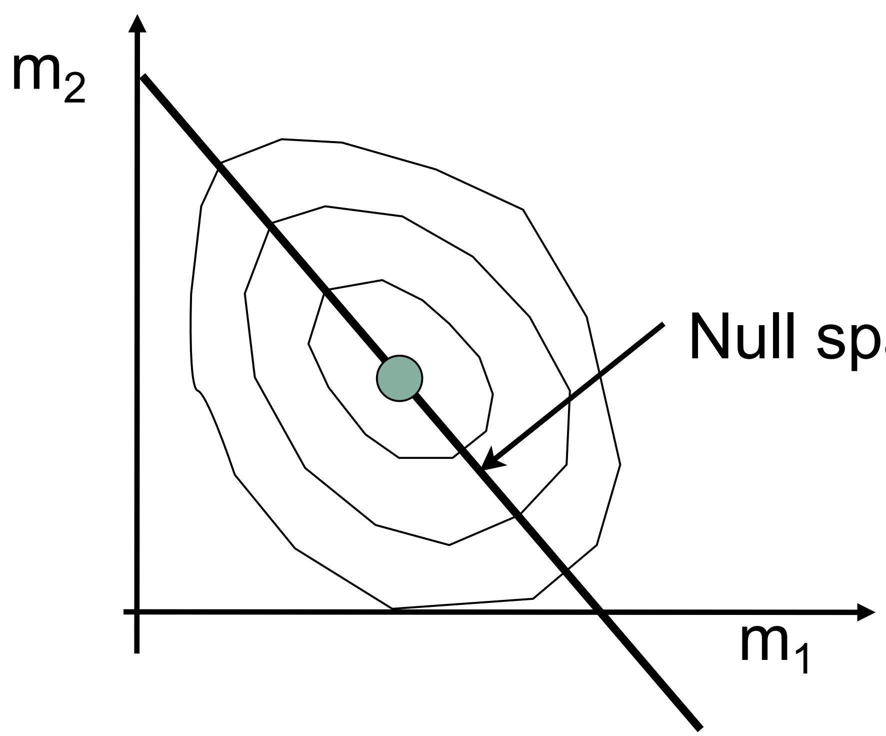
Newton or Ptolemy?

There exists sophisticated Bayesian model-selection procedures for general nonlinear regression problems.

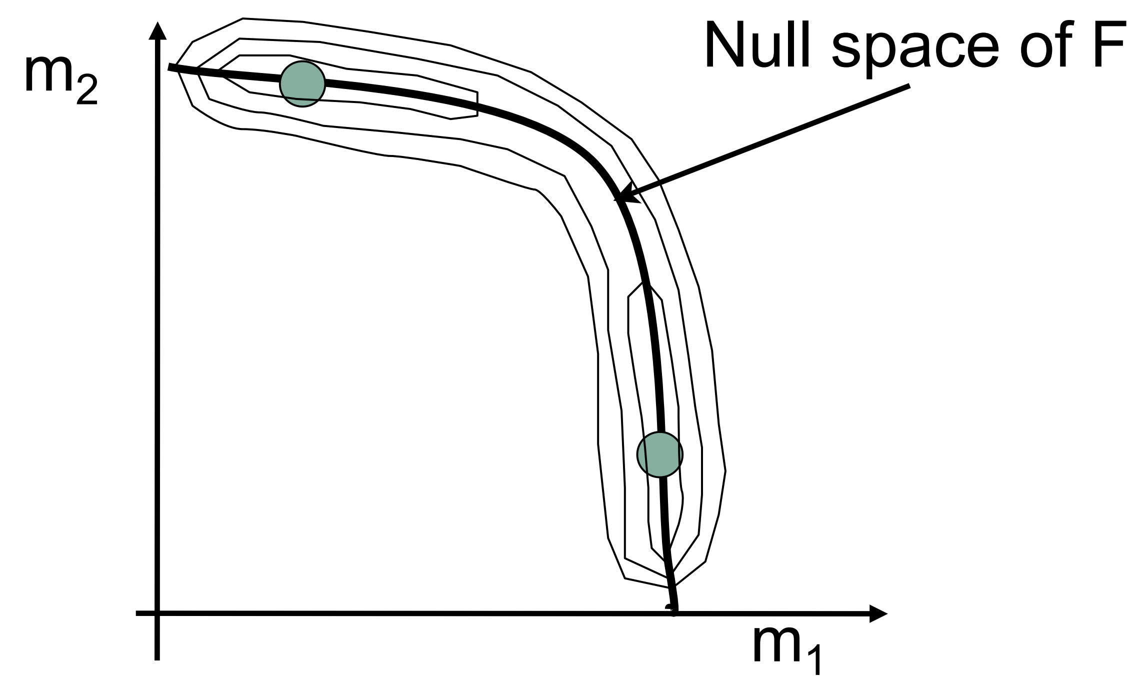
Tight & non-linear -- an inversion difficulty

$$\chi^2 = (\mathbf{d} - \mathbf{F}(\mathbf{m}))^T C_d^{-1} (\mathbf{d} - \mathbf{F}(\mathbf{m})) + (\mathbf{m} - \mathbf{m}_p)^T C_p^{-1} (\mathbf{m} - \mathbf{m}_p)$$

seismic case



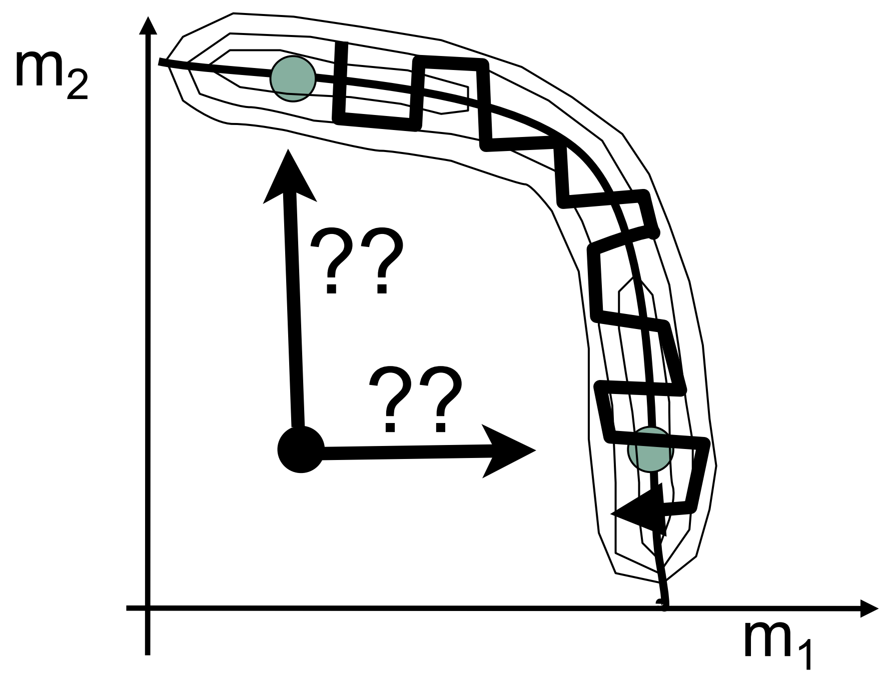
CSEM case



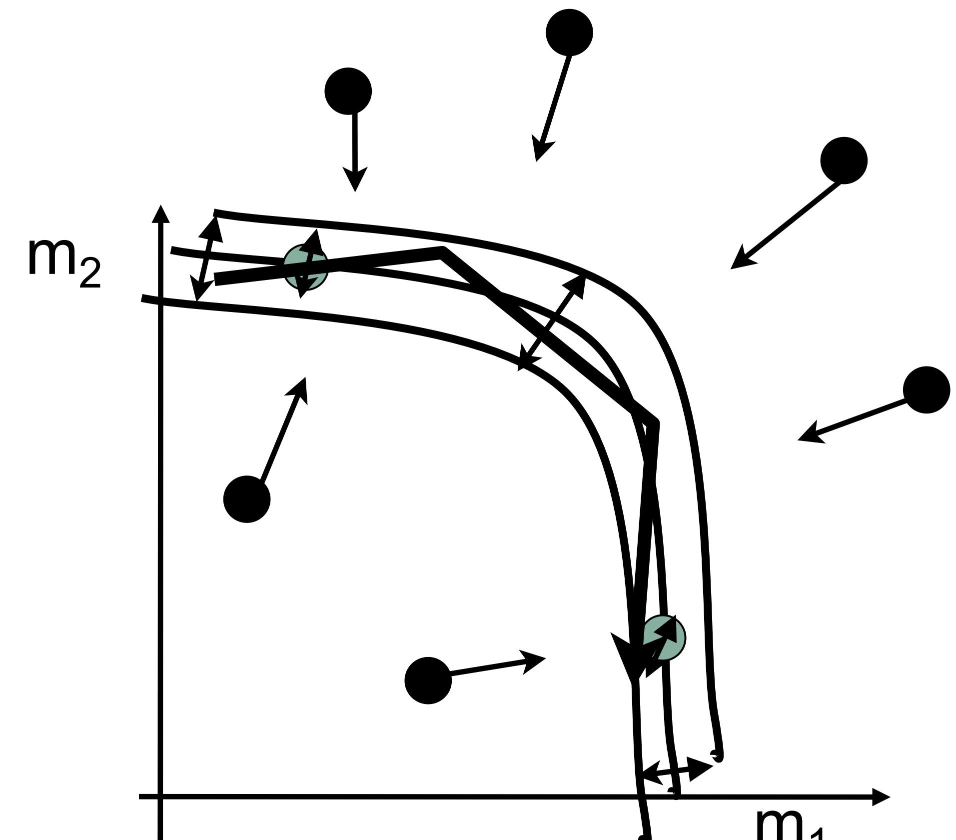
no Born approximation because of strong contrasts; therefore multimodal, badly scaled, and contorted in shape

The problem and the solution

problem

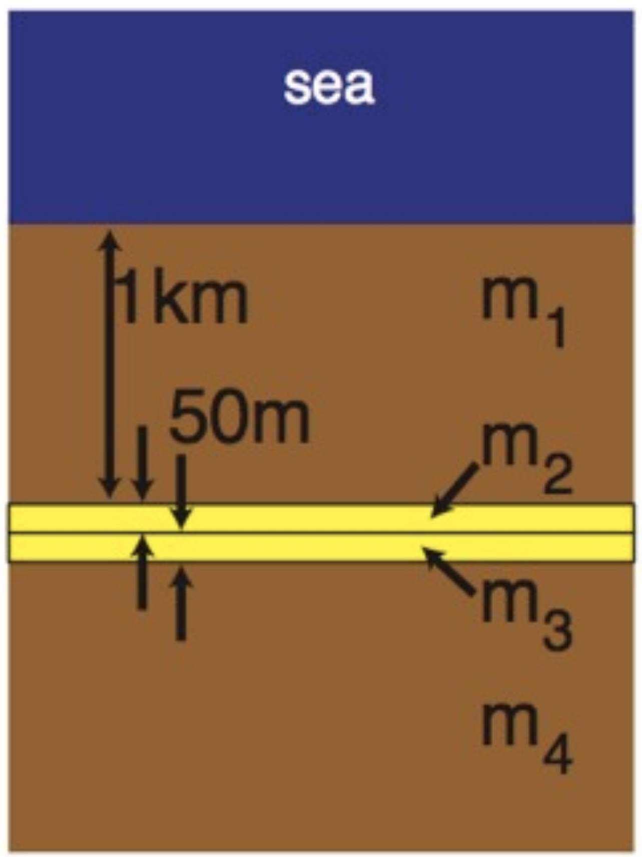


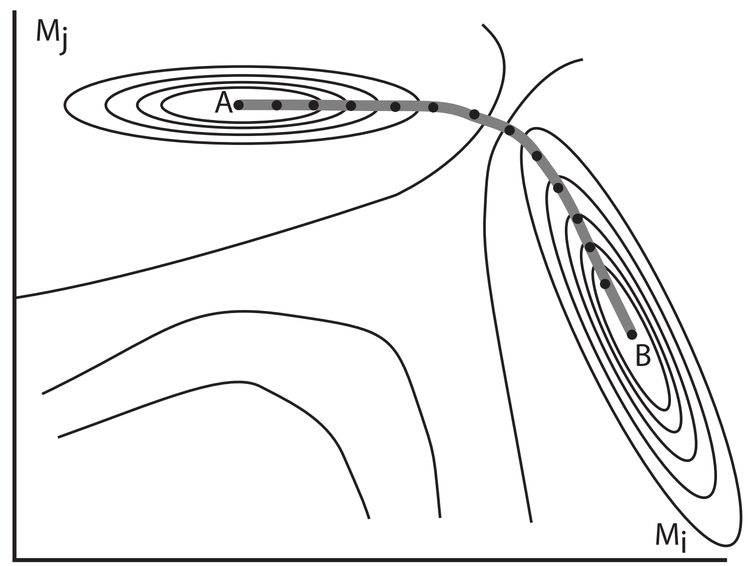
solutions

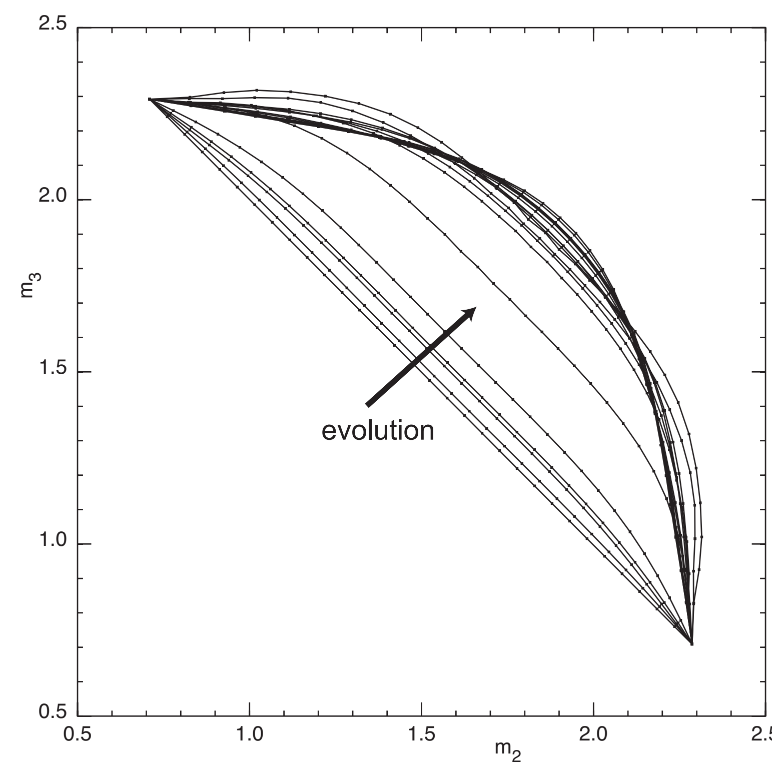


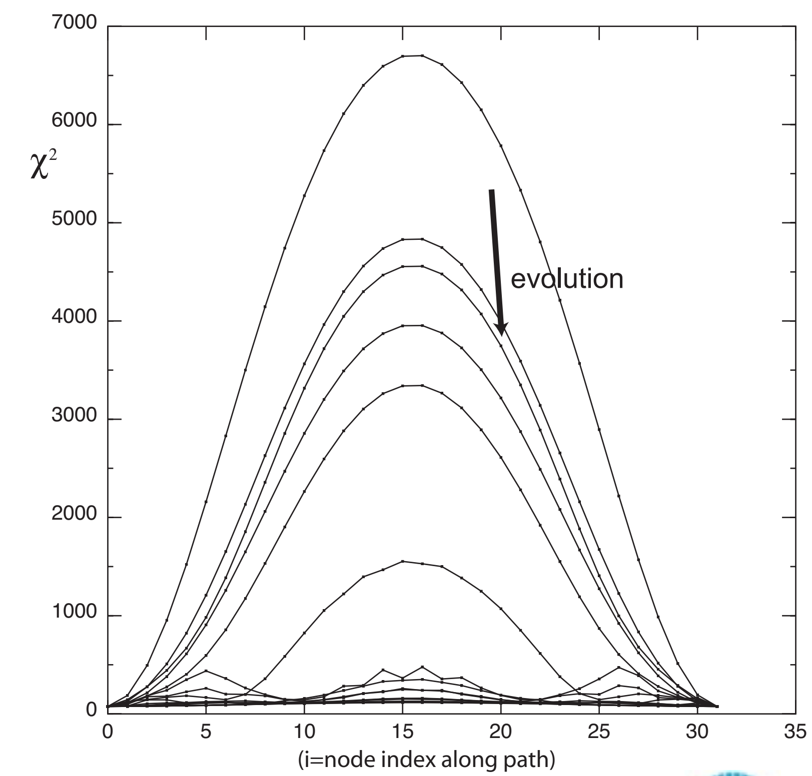
1. Parametric bootstrap
   multiple start optimization
   perturb data with noise
2. Change coordinates to mode connection paths

Mode connection paths

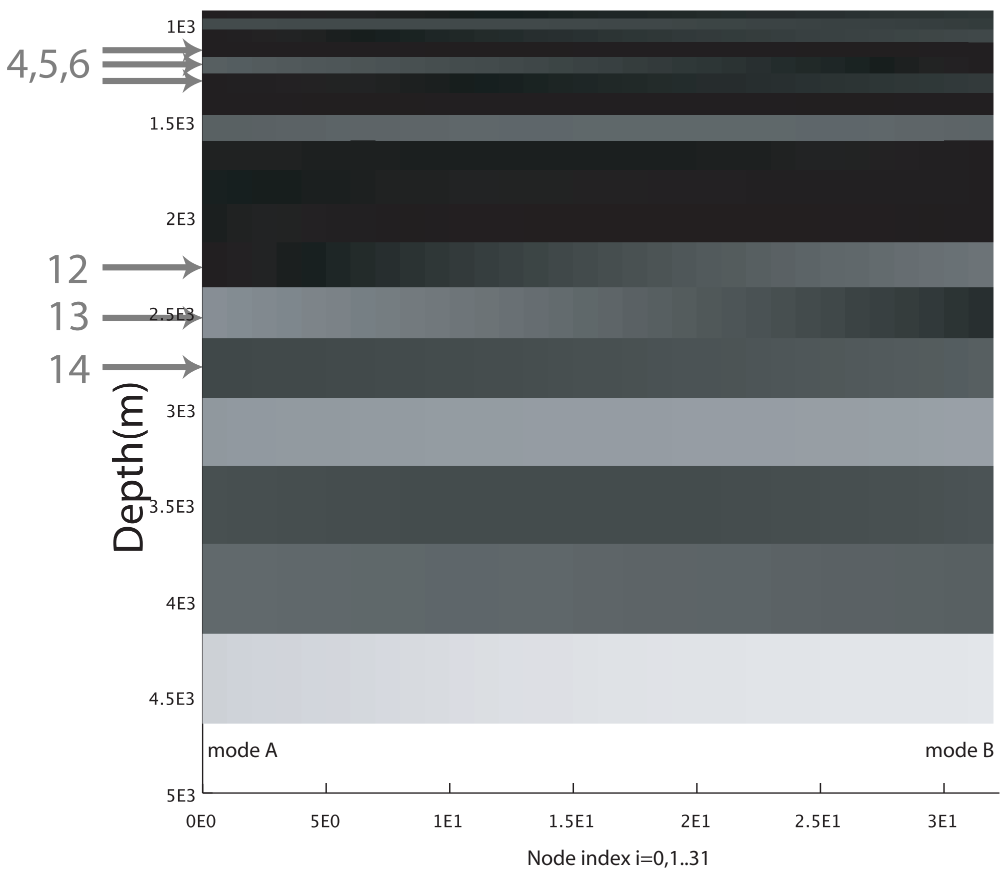


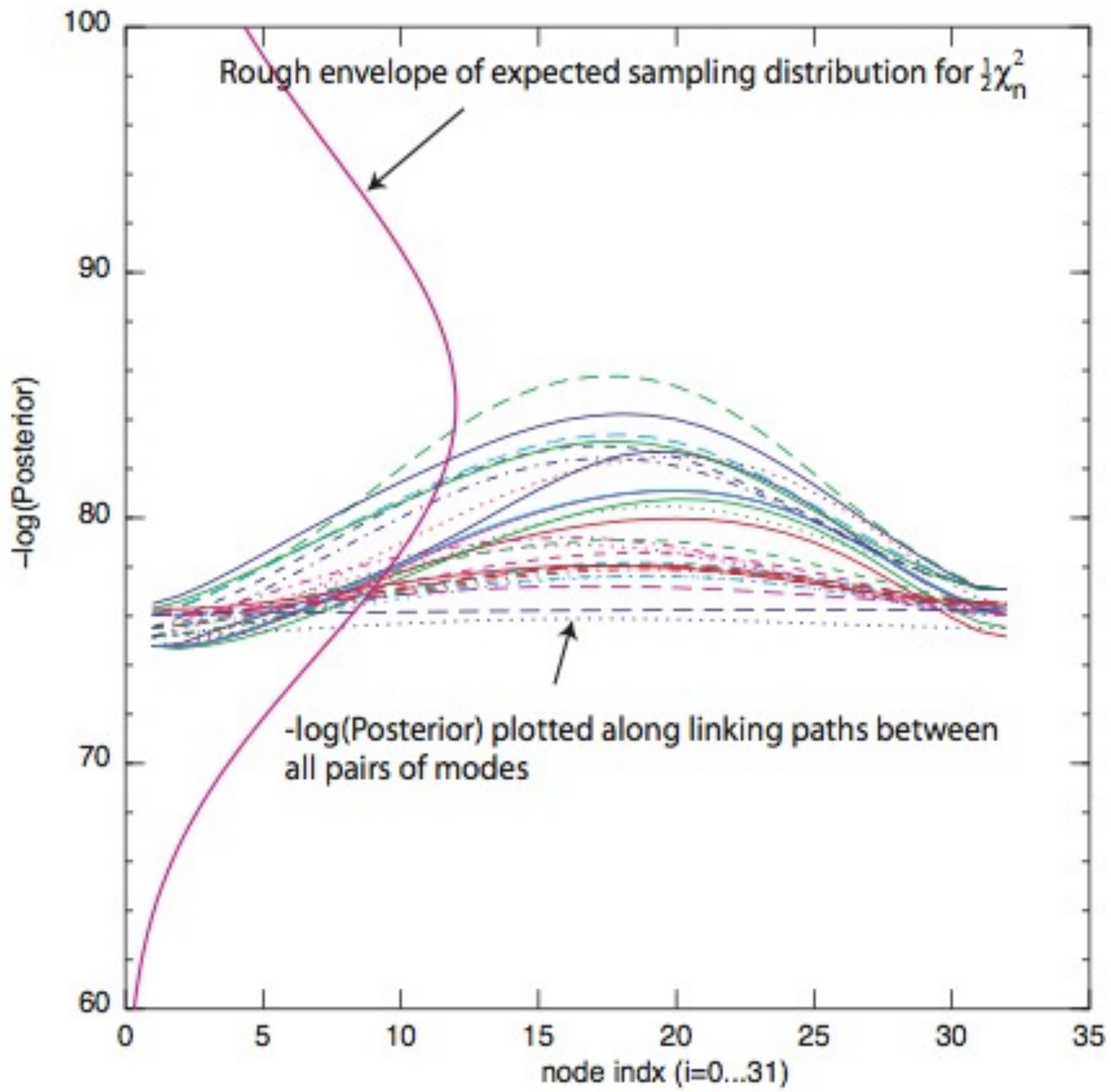




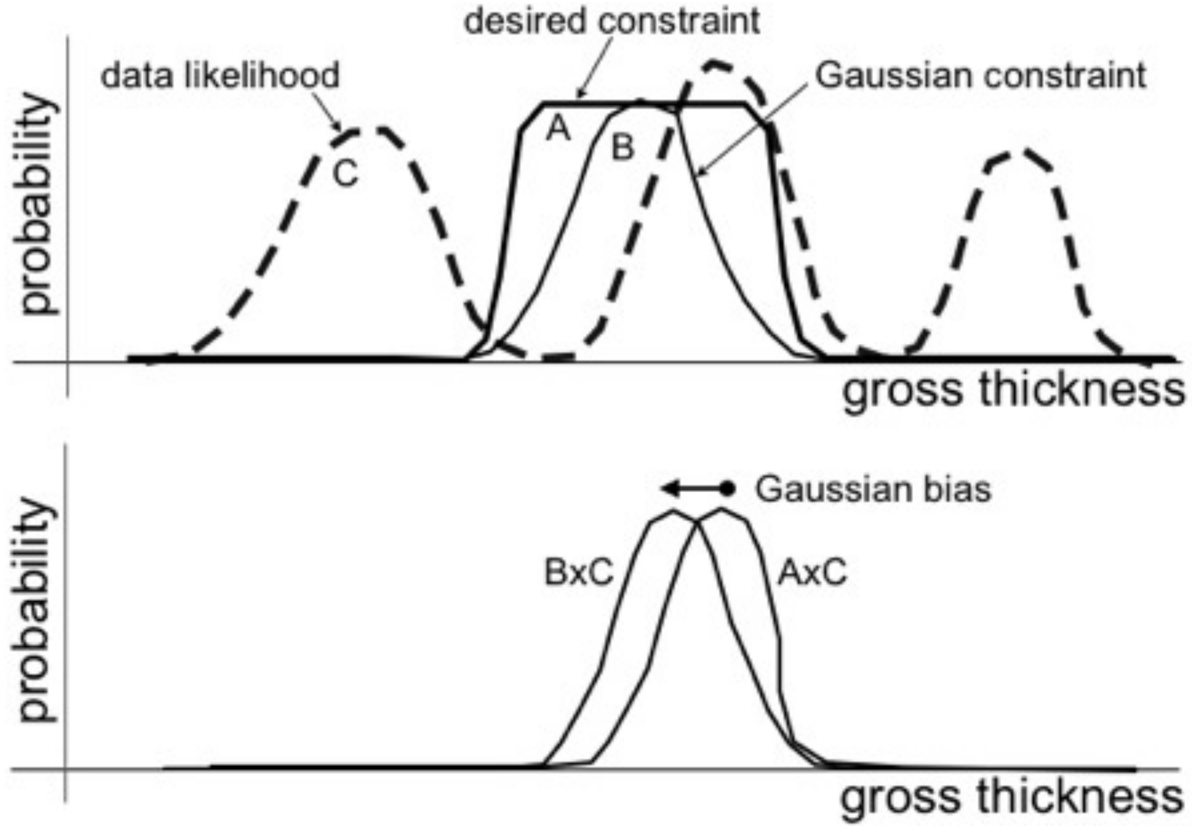


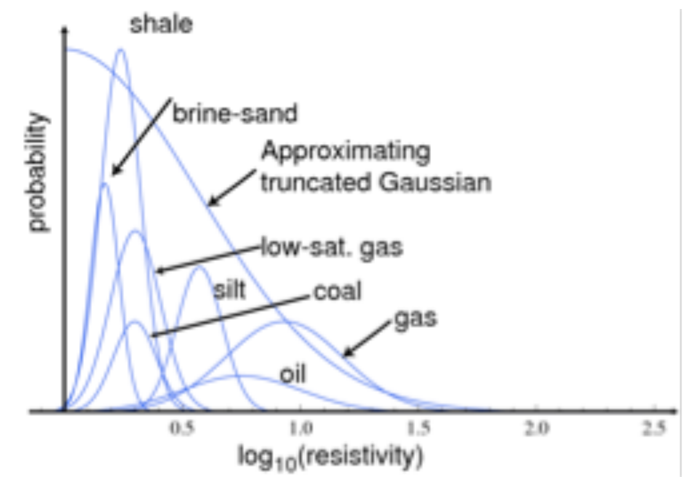
An more realistic example of mode connection paths

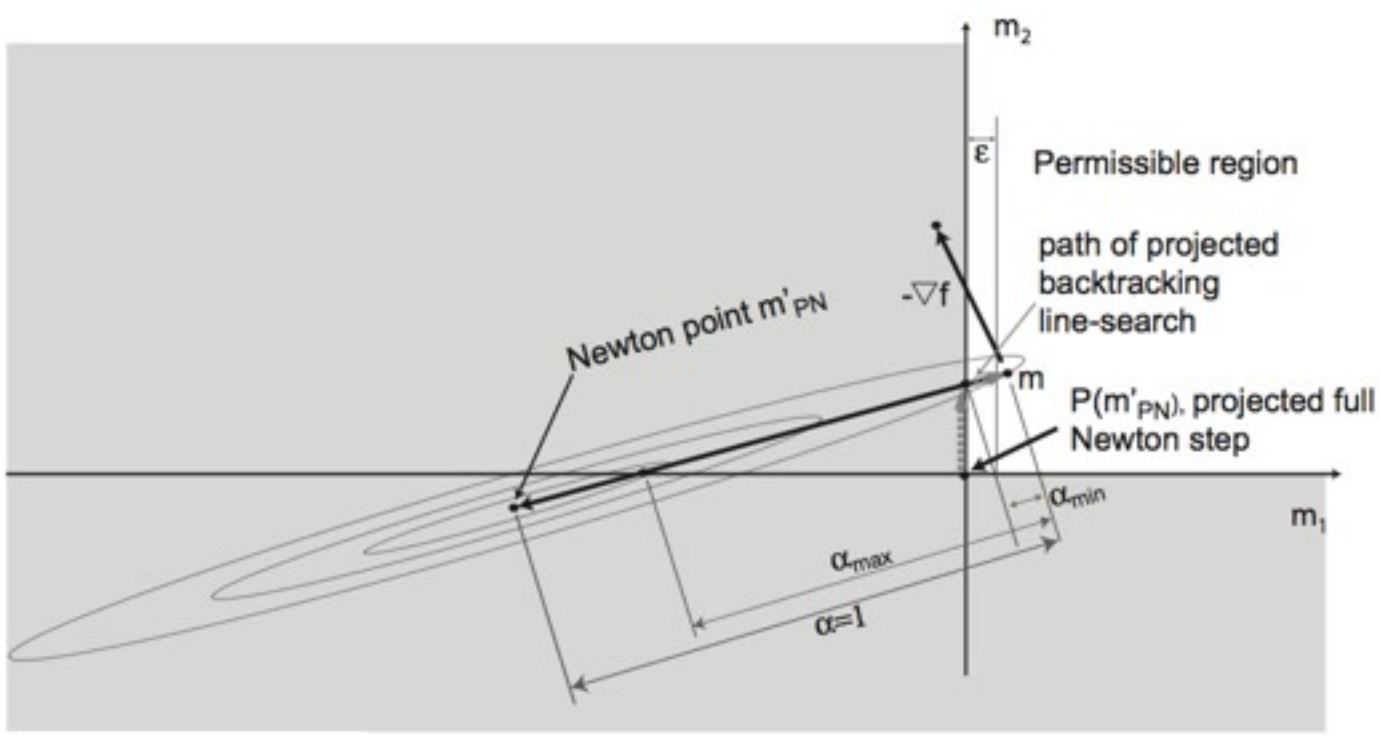




Truncated distributions -- an inversion difficulty







Connections to geostatistics

If $C_{p,ij} = \sigma_p^2 \exp(-|i-j|/\lambda_p)$

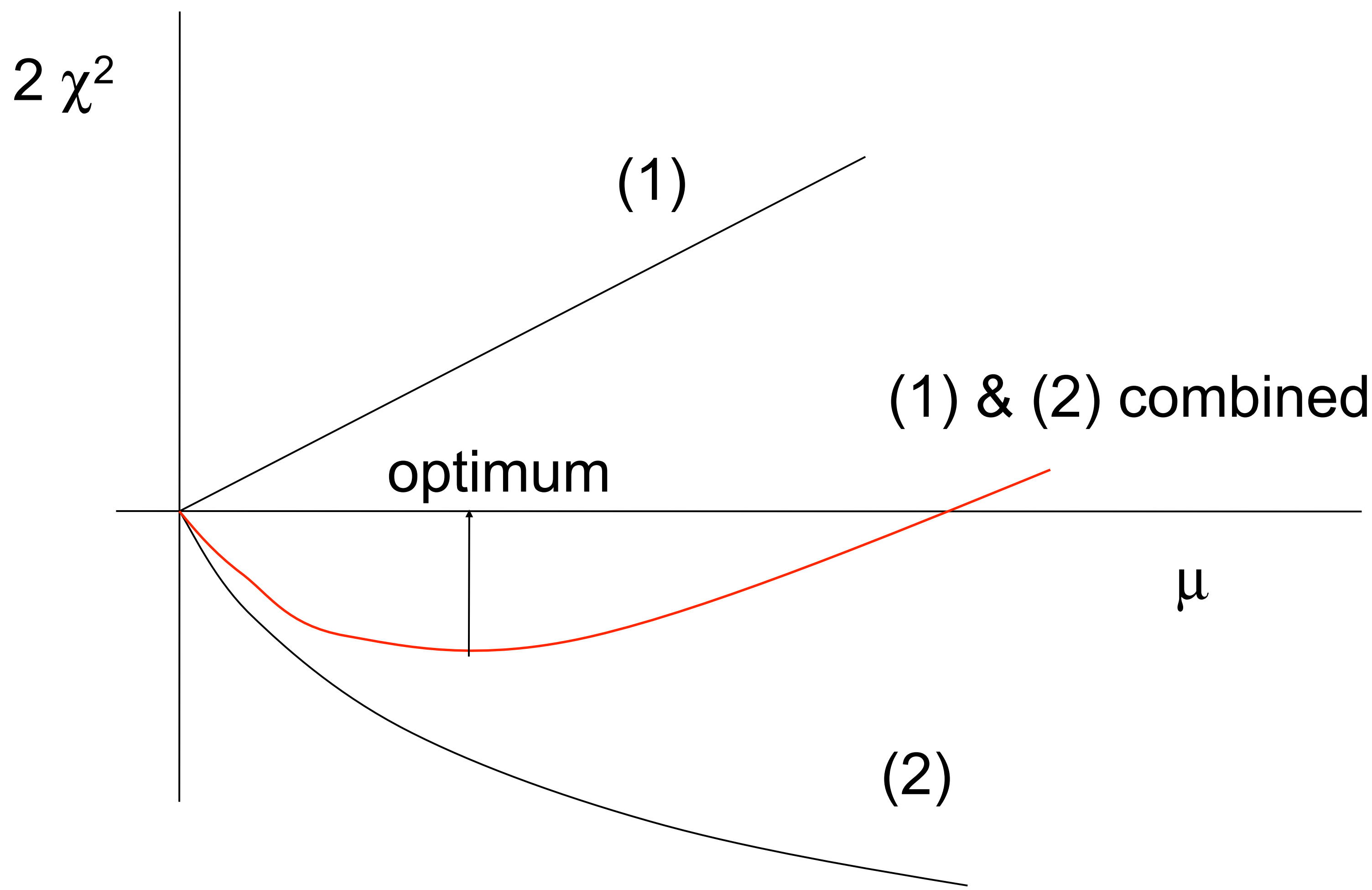
$$
\begin{aligned}
C_p^{-1} &= \sigma_p^{-2} \begin{pmatrix} e_1 & e_2 & 0 & 0 & 0 & \dots \\ e_2 & e_3 & e_2 & 0 & 0 & \dots \\ 0 & e_2 & e_3 & e_2 & 0 & \dots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & e_2 & e_1 \end{pmatrix} \\
e_1 &= 1/(1-e^{-2/\lambda_p}) \\
e_2 &= -e^{-1/\lambda_p}/(1-e^{-2/\lambda_p}) \\
e_3 &= (1+e^{-2/\lambda_p})/(1-e^{-2/\lambda_p}) \\
|C_p| &= \sigma_p^{2n_m}(1-e^{-2/\lambda_p})^{n_m-1}
\end{aligned}
$$

Degeneracy: $\log|C_p| \to -\infty$ as $\lambda_p \to \infty$

Bayesian smoothing formulation

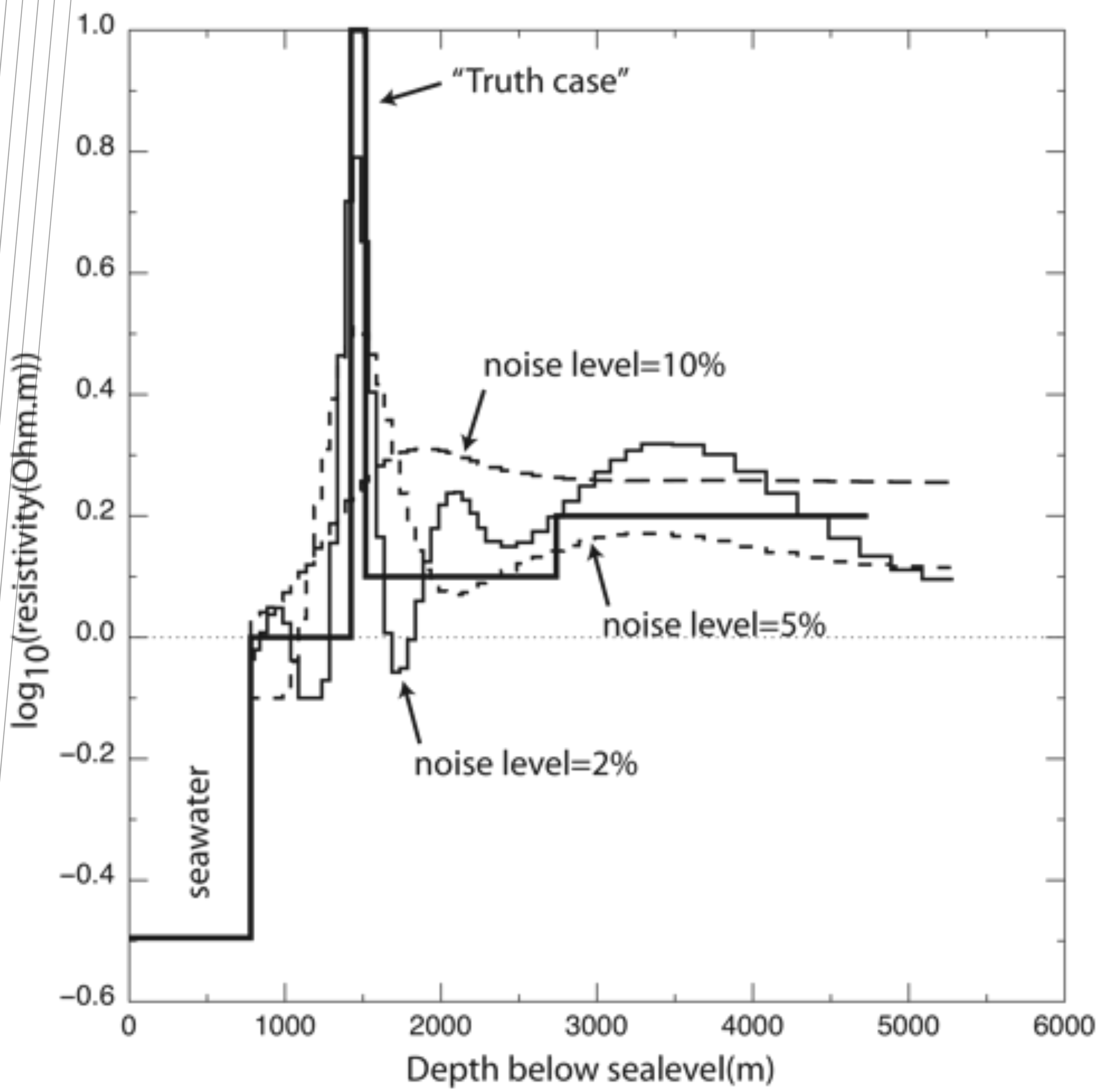
$$
\begin{aligned}
2\chi^2 &= (\mathbf{d}-\mathbf{F}(\mathbf{m}))^T C_d^{-1}(\mathbf{d}-\mathbf{F}(\mathbf{m})) + (\mathbf{m}-\mathbf{m}_p)^T C_p(m')^{-1}(\mathbf{m}-\mathbf{m}_p) + \log|C_p(m')| \\
&= (\mathbf{d}-\mathbf{F}(\mathbf{m}))^T C_d^{-1}(\mathbf{d}-\mathbf{F}(\mathbf{m})) \\
&\quad + \underbrace{\mu(\mathbf{m}-\mathbf{m}_p)^T(\partial^T\partial)^{-1}(\mathbf{m}-\mathbf{m}_p)}_{(1)} + (\mathbf{m}-\mathbf{m}_p)^T C_{p,\mathrm{diag}}^{-1}(\mathbf{m}-\mathbf{m}_p) \underbrace{- \log|\mu(\partial^T\partial)^{-1} + C_{p,\mathrm{diag}}(\mu)|}_{(2)}
\end{aligned}
$$

New piece

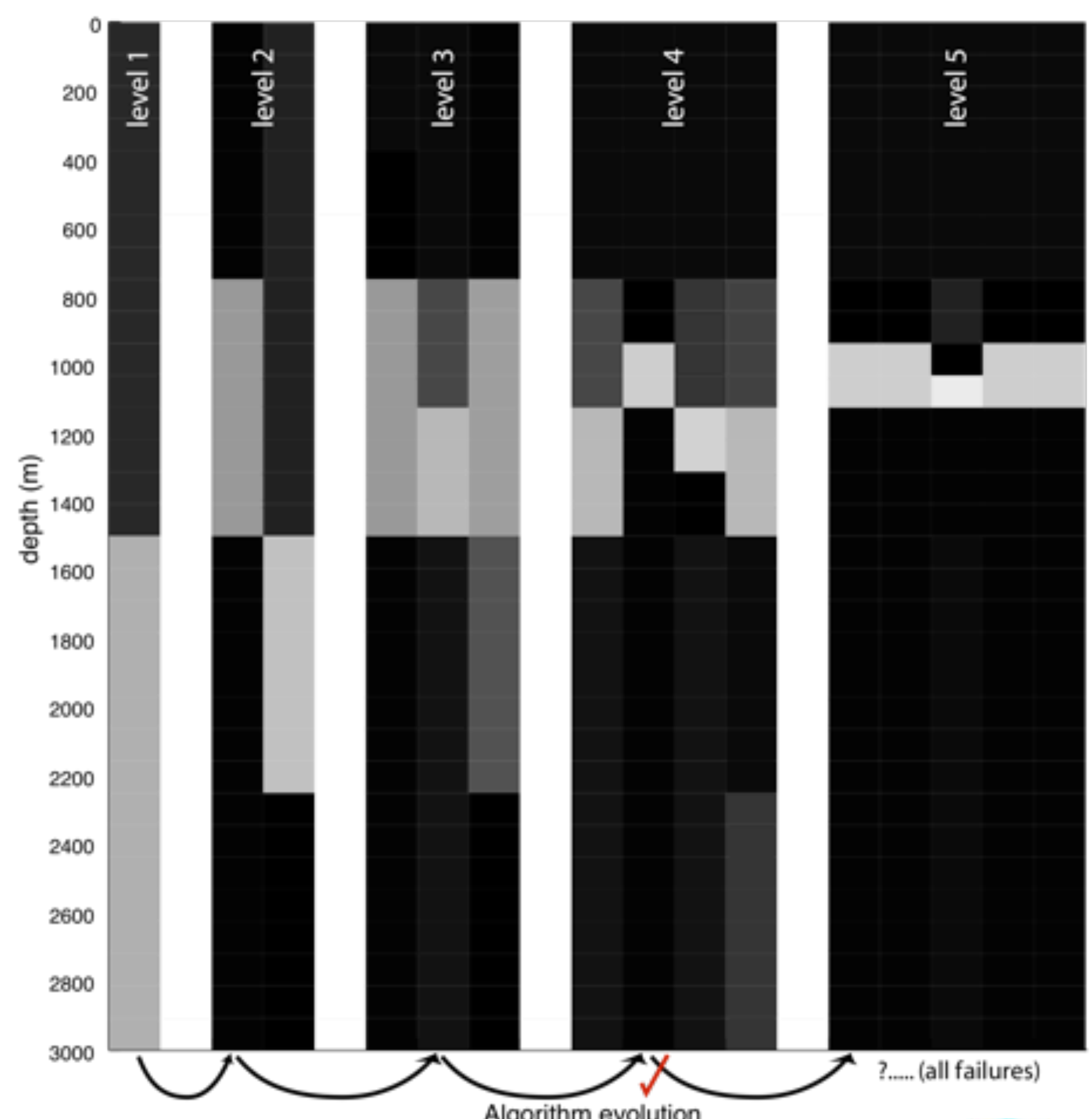


Two flavors of Bayesian inversion

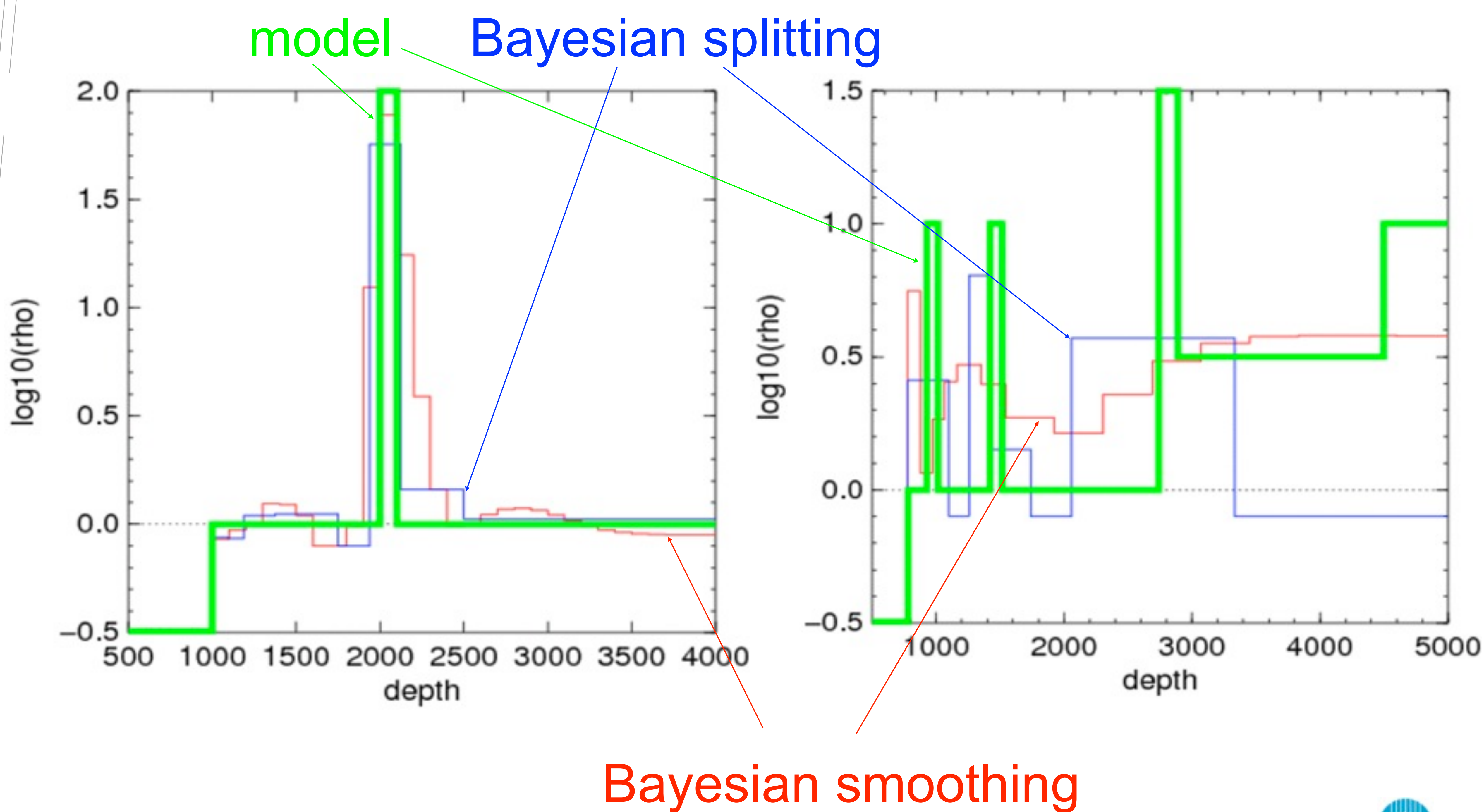
Bayesian Smoothing



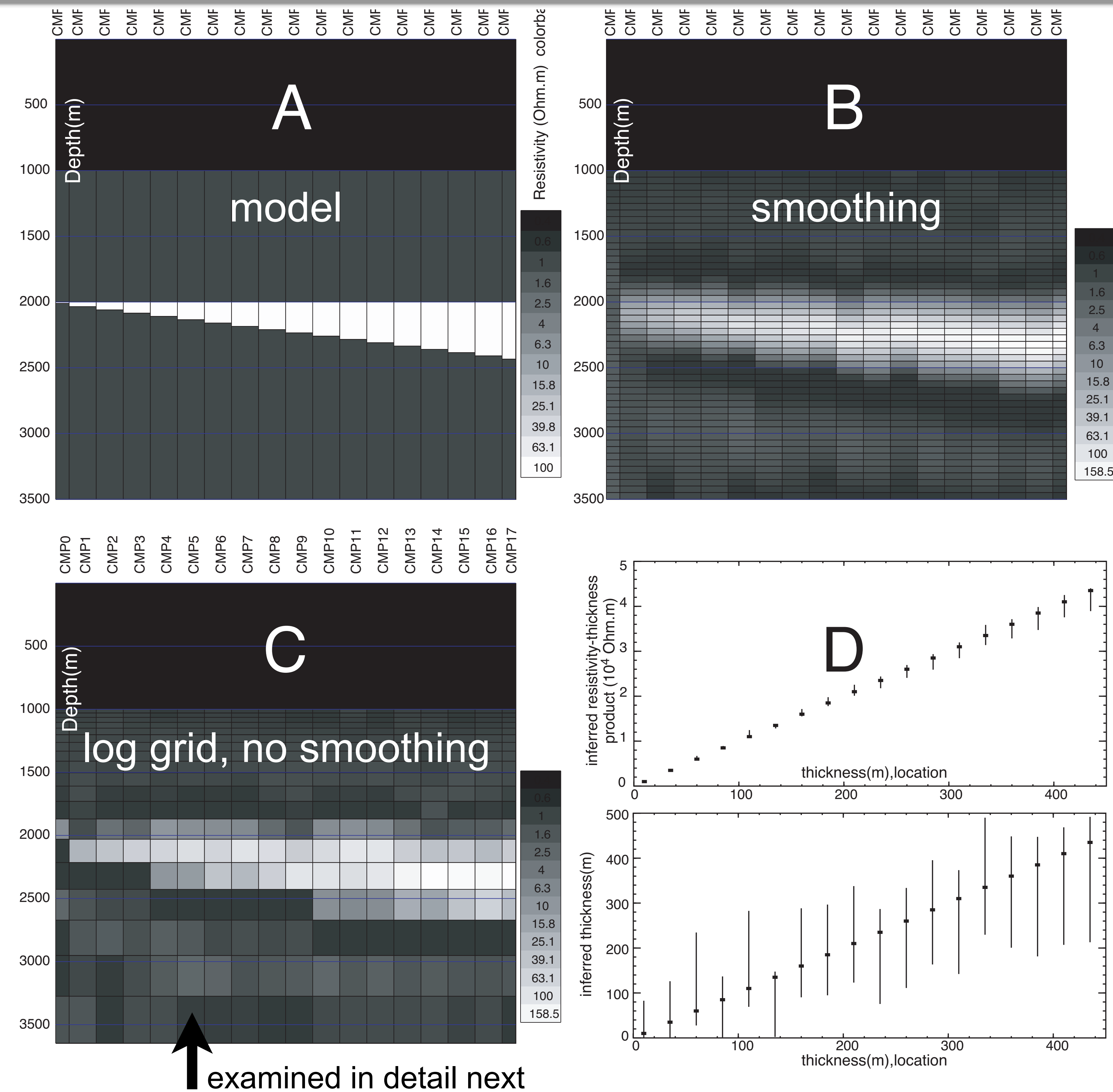
Bayesian model-selection (splitting/merging)



Comparison of the two flavors

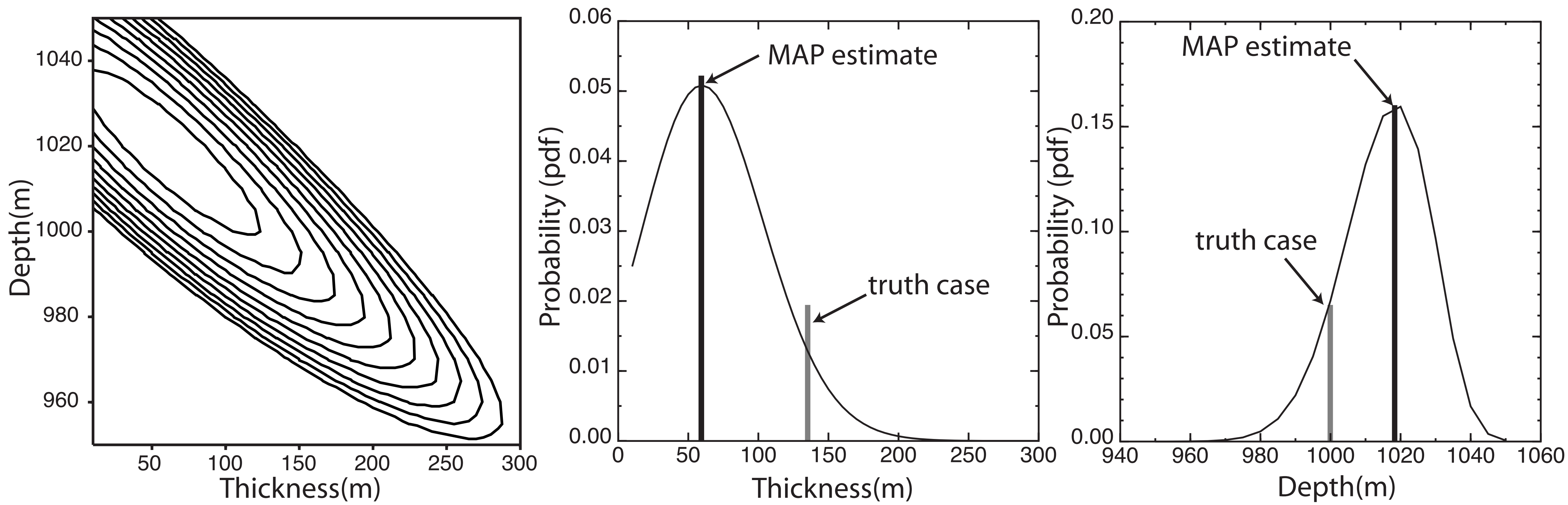


Wedge model

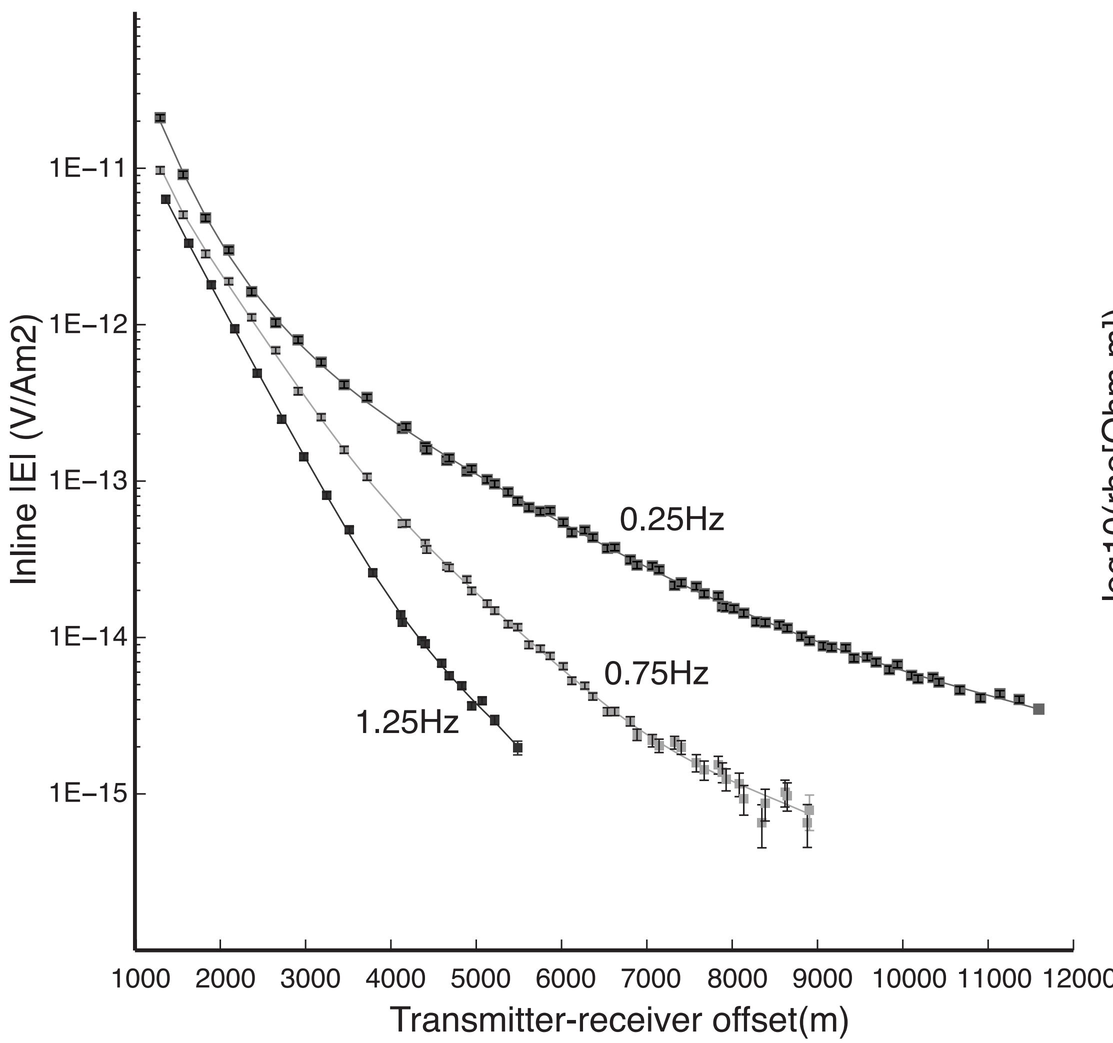


A
model
Depth(m)
Resistivity (Ohm.m) colorba
B
smoothing
Depth(m)
C
log grid, no smoothing
Depth(m)
CMP0 CMP1 CMP2 CMP3 CMP4 CMP5 CMP6 CMP7 CMP8 CMP9 CMP10 CMP11 CMP12 CMP13 CMP14 CMP15 CMP16 CMP17
D
inferred resistivity-thickness product (10^4 Ohm.m)
thickness(m),location
inferred thickness(m)
thickness(m),location
examined in detail next

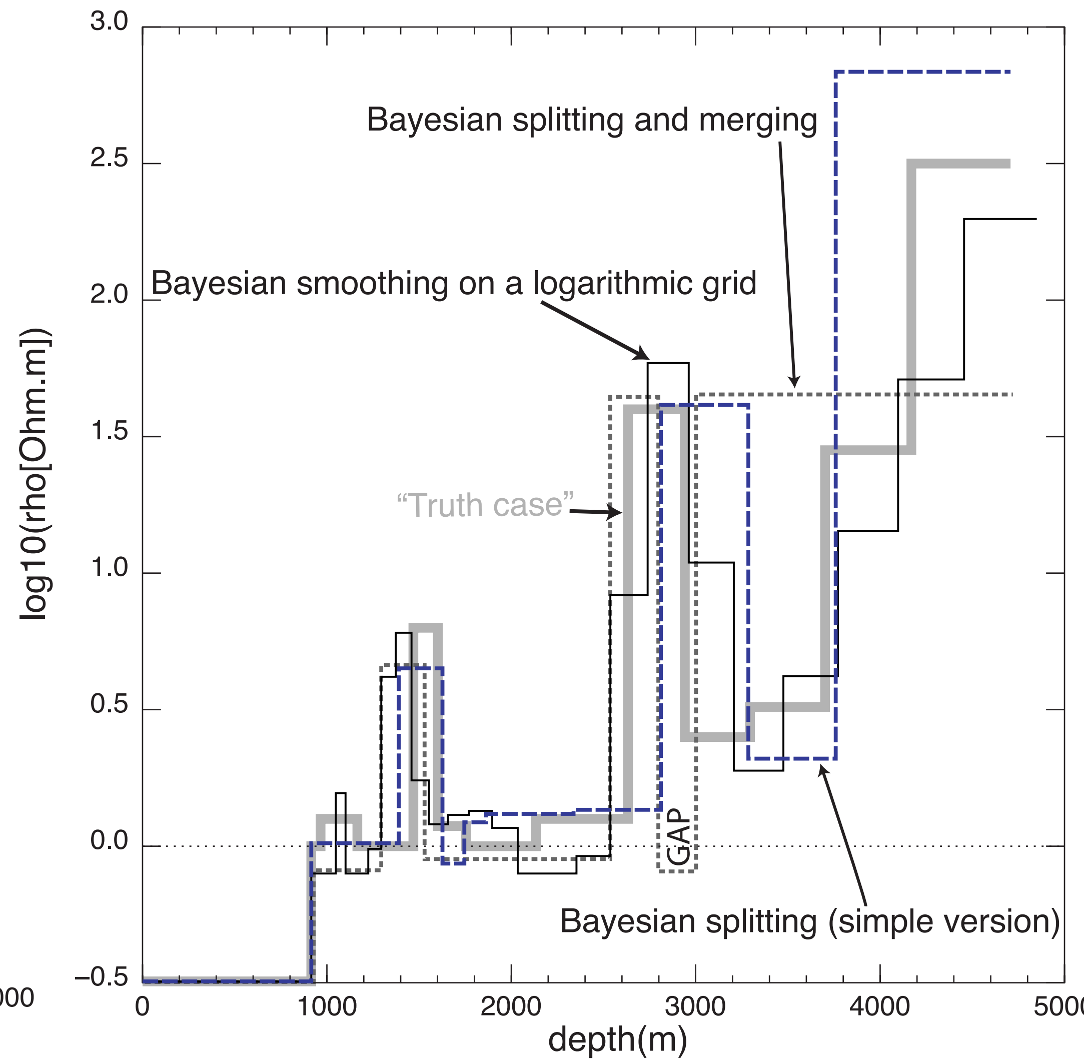
Thickness and depth resolution



“Bird” model



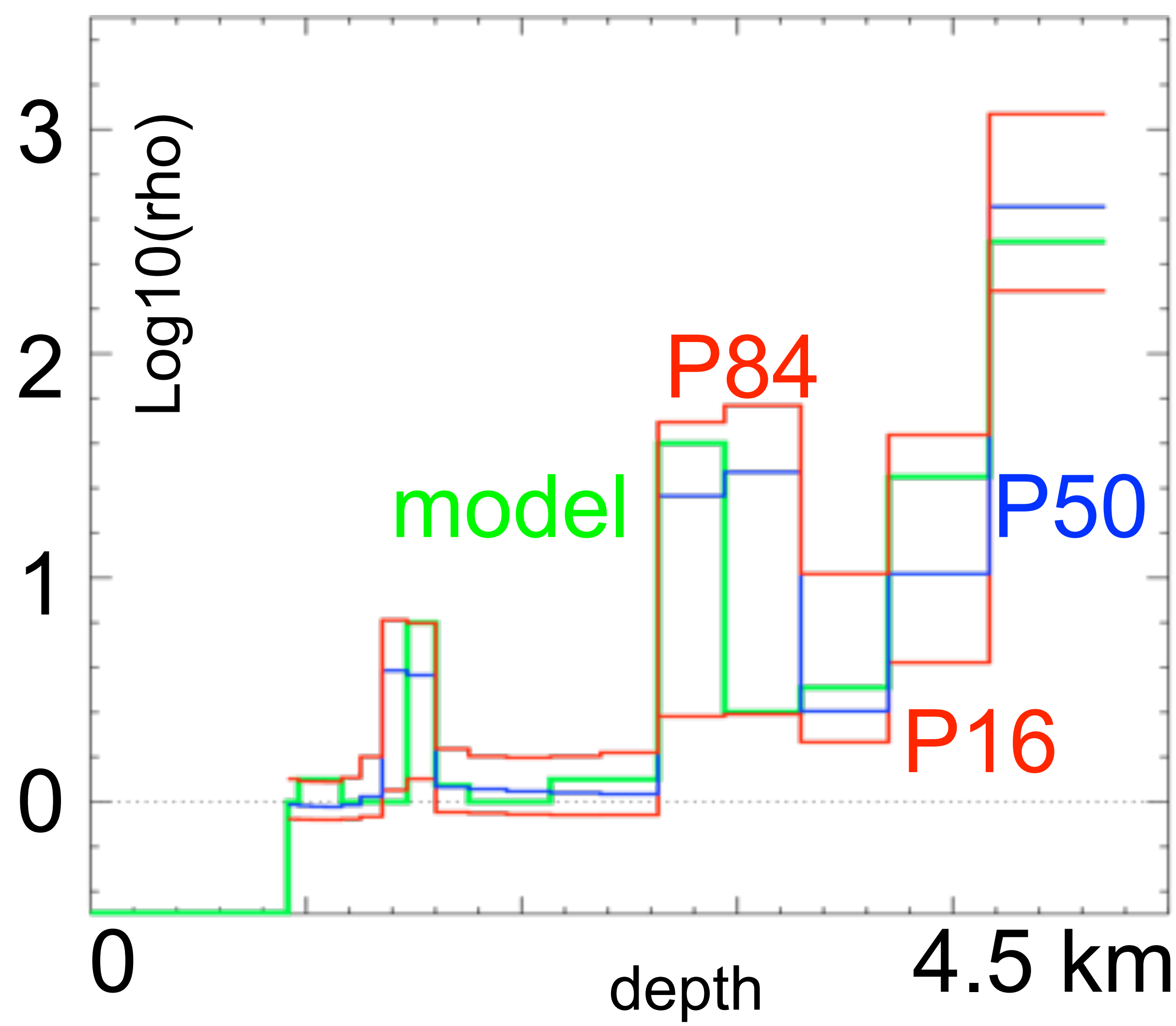
Inline IEI (V/Am2)
1E−11
1E−12
1E−13
1E−14
1E−15
0.25Hz
0.75Hz
1.25Hz
1000 2000 3000 4000 5000 6000 7000 8000 9000 10000 11000 12000
Transmitter-receiver offset(m)



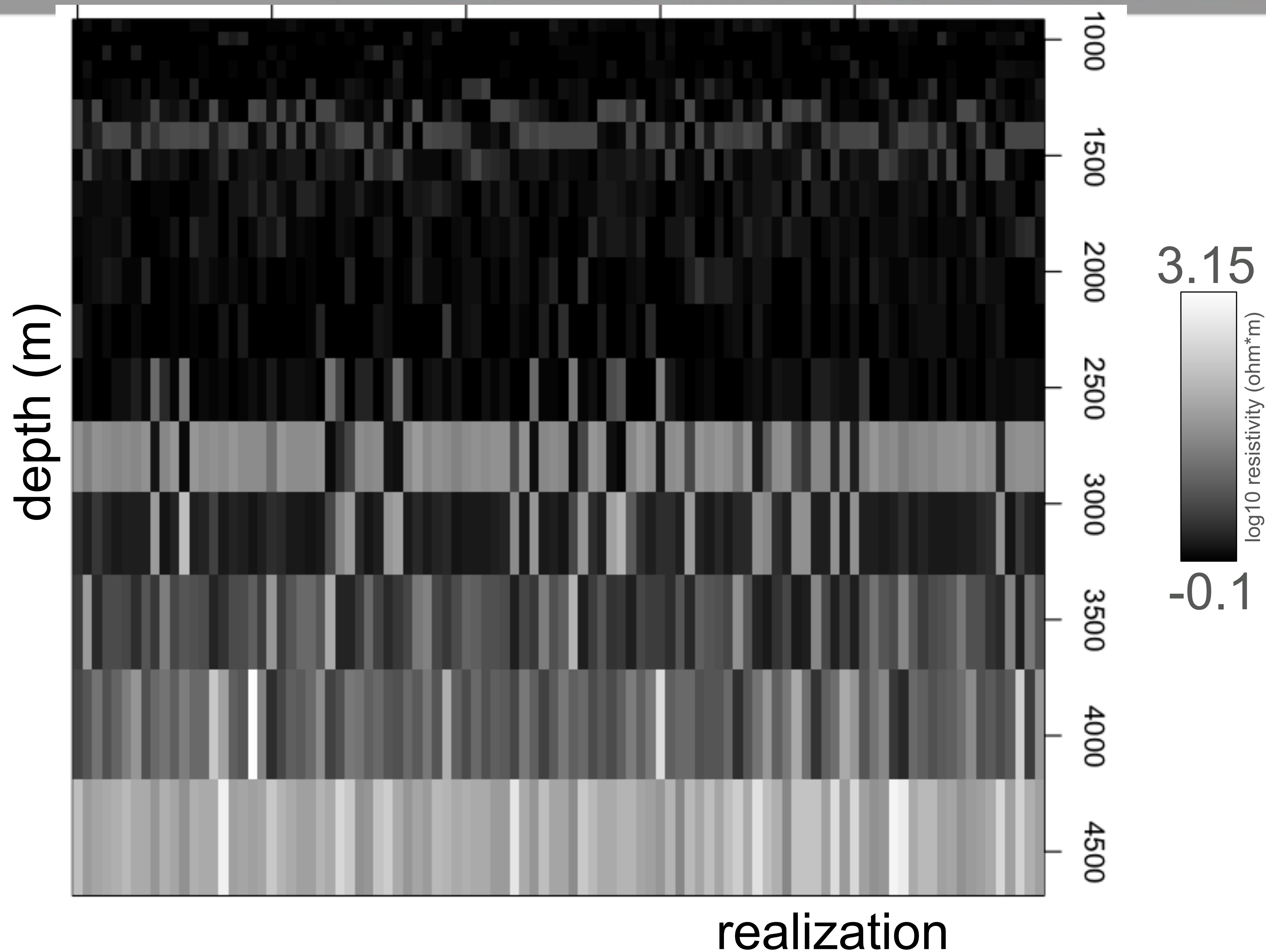
log10(rho[Ohm.m])
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
Bayesian splitting and merging
Bayesian smoothing on a logarithmic grid
“Truth case”
GAP
Bayesian splitting (simple version)
0 1000 2000 3000 4000 5000
depth(m)

Uncertainty sampling methods for “bird” model

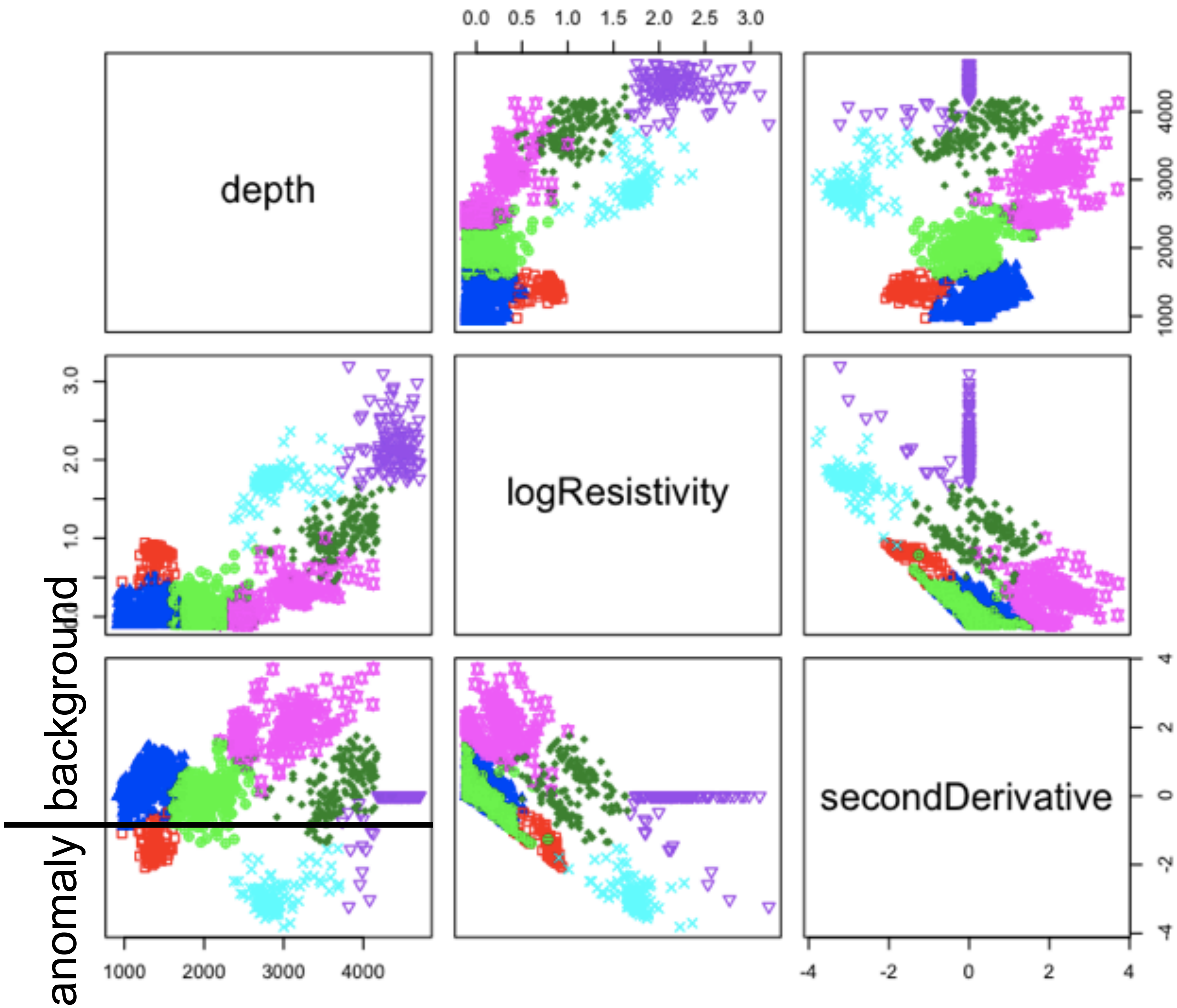
- Local covariances pretty hopeless
- Sampling methods only possibility
  - MCMC : OK but very demanding (narrow twisty objective)
  - Bayesianized parametric bootstrap: approximate method which is fairly good, uses optimization methods very heavily



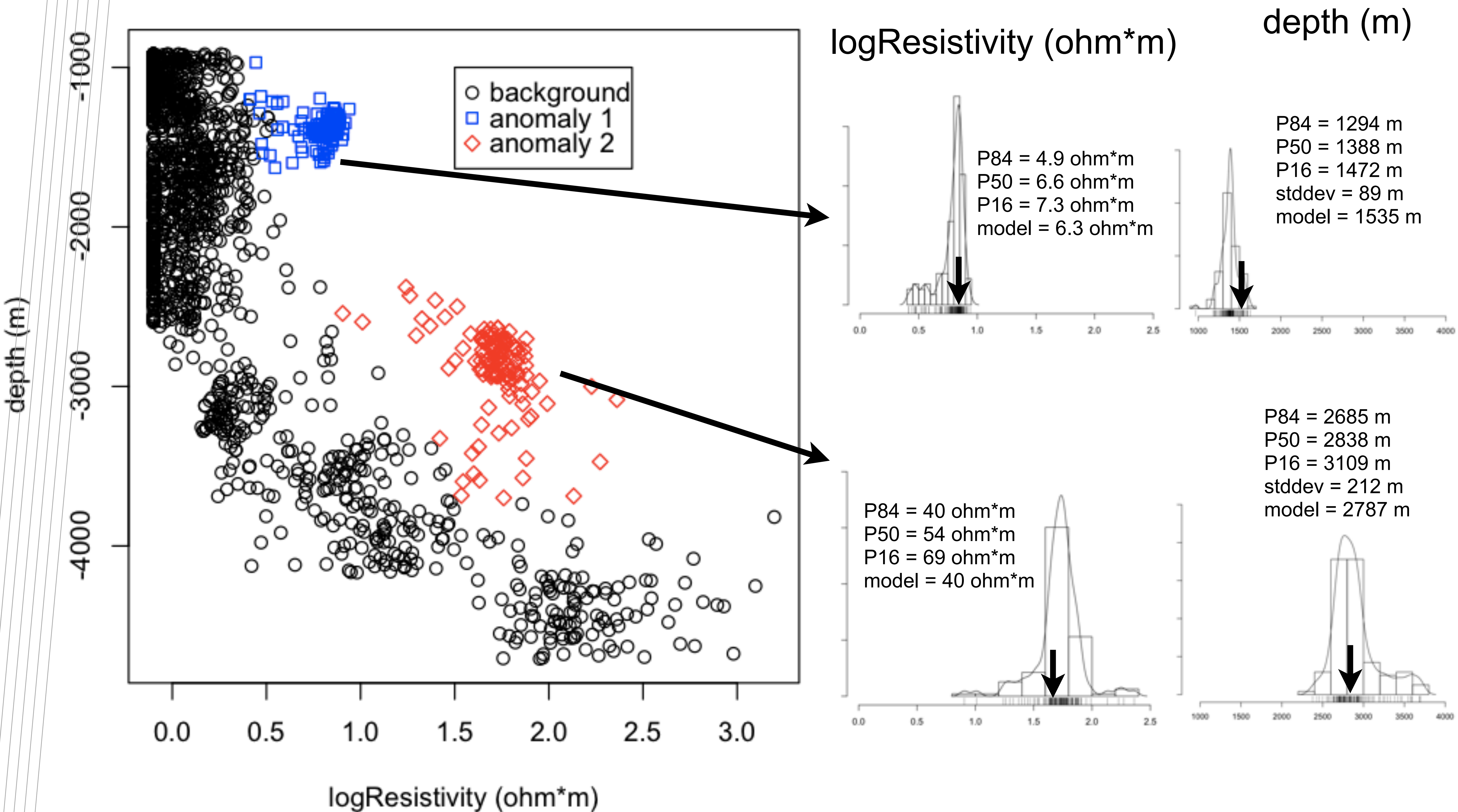
Log resistivity profiles for individual realizations



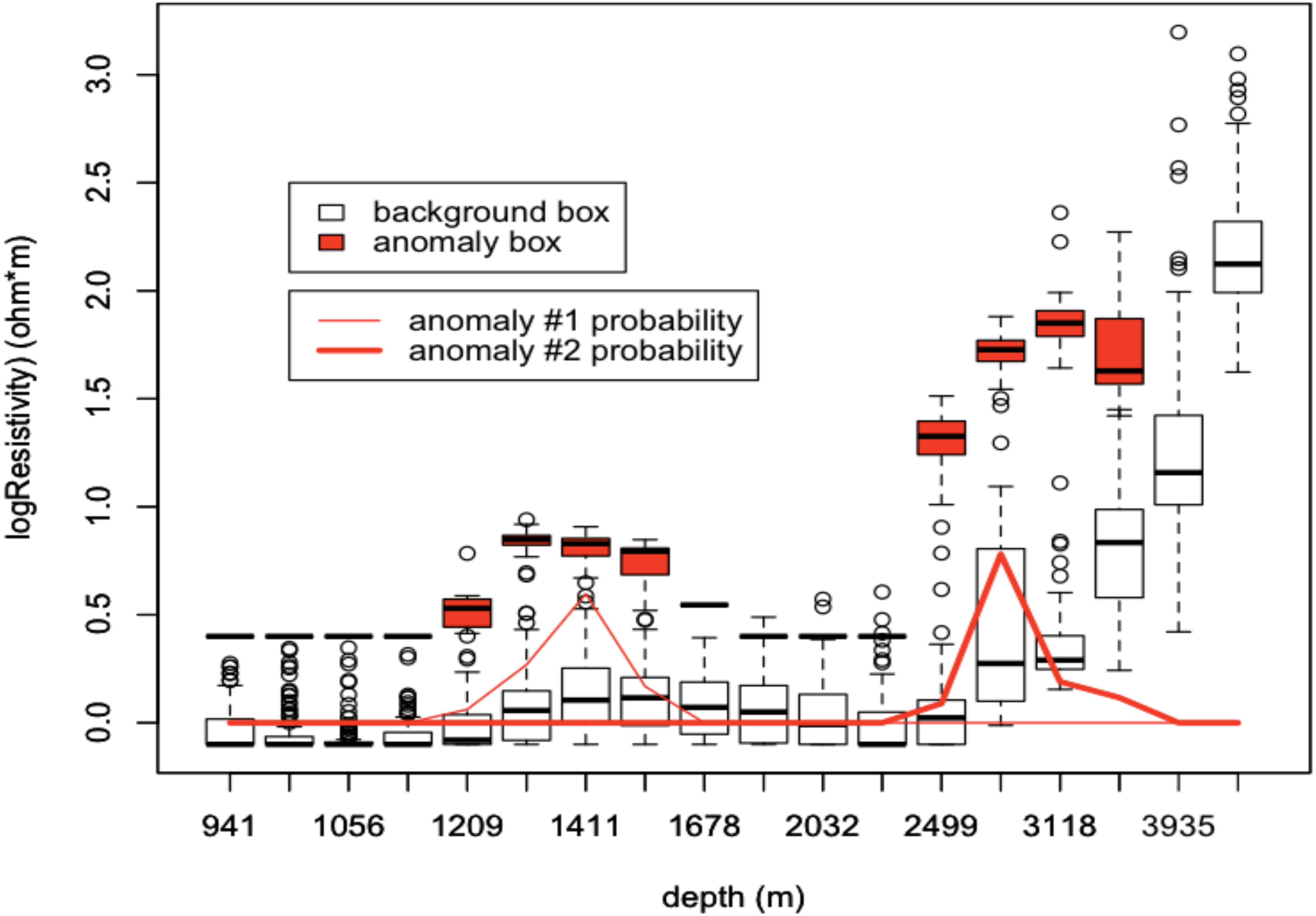
Cluster separation to discriminate background from anomaly



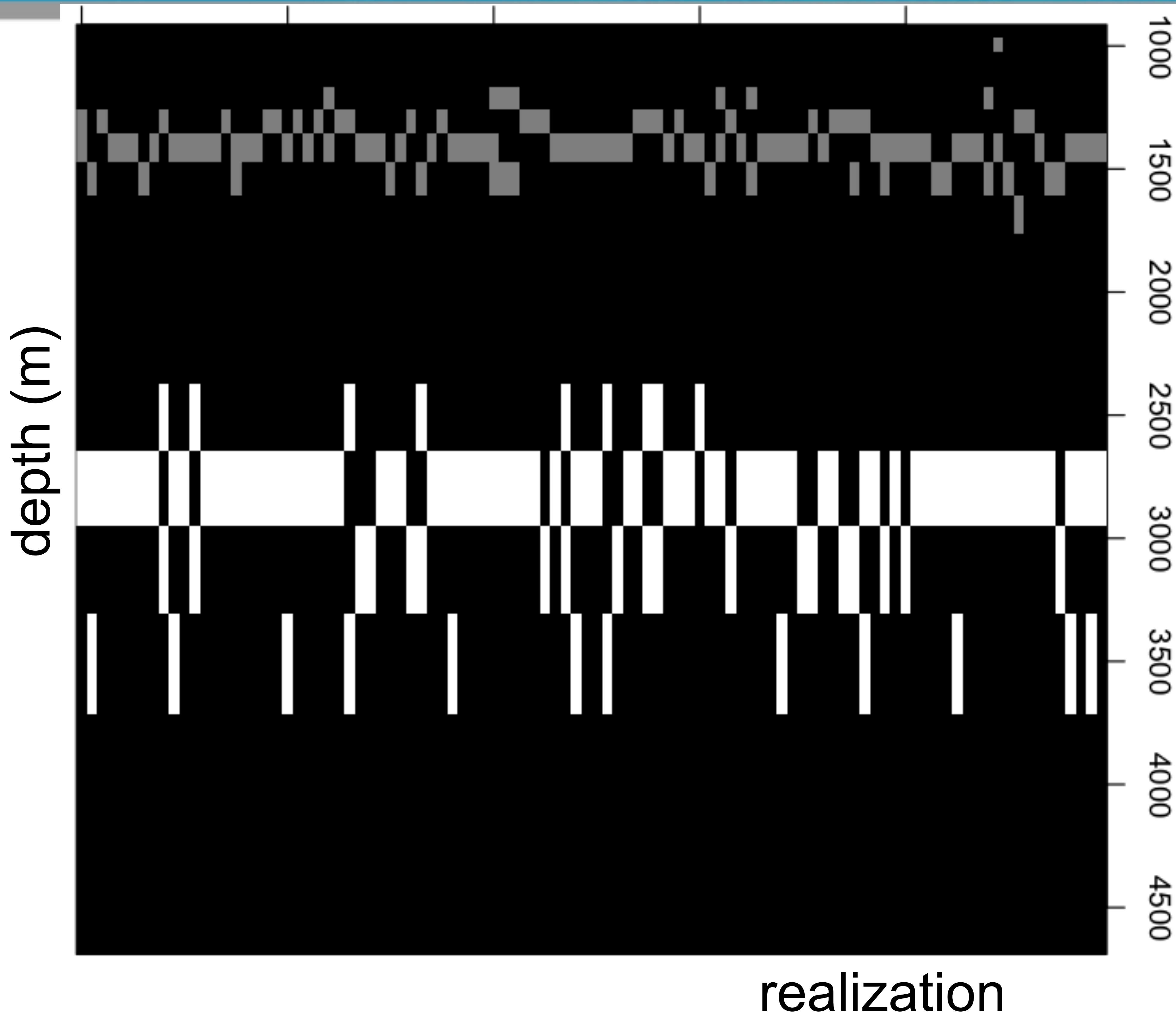
Individual anomaly cluster distributions



Summary of individual anomaly clusters and background



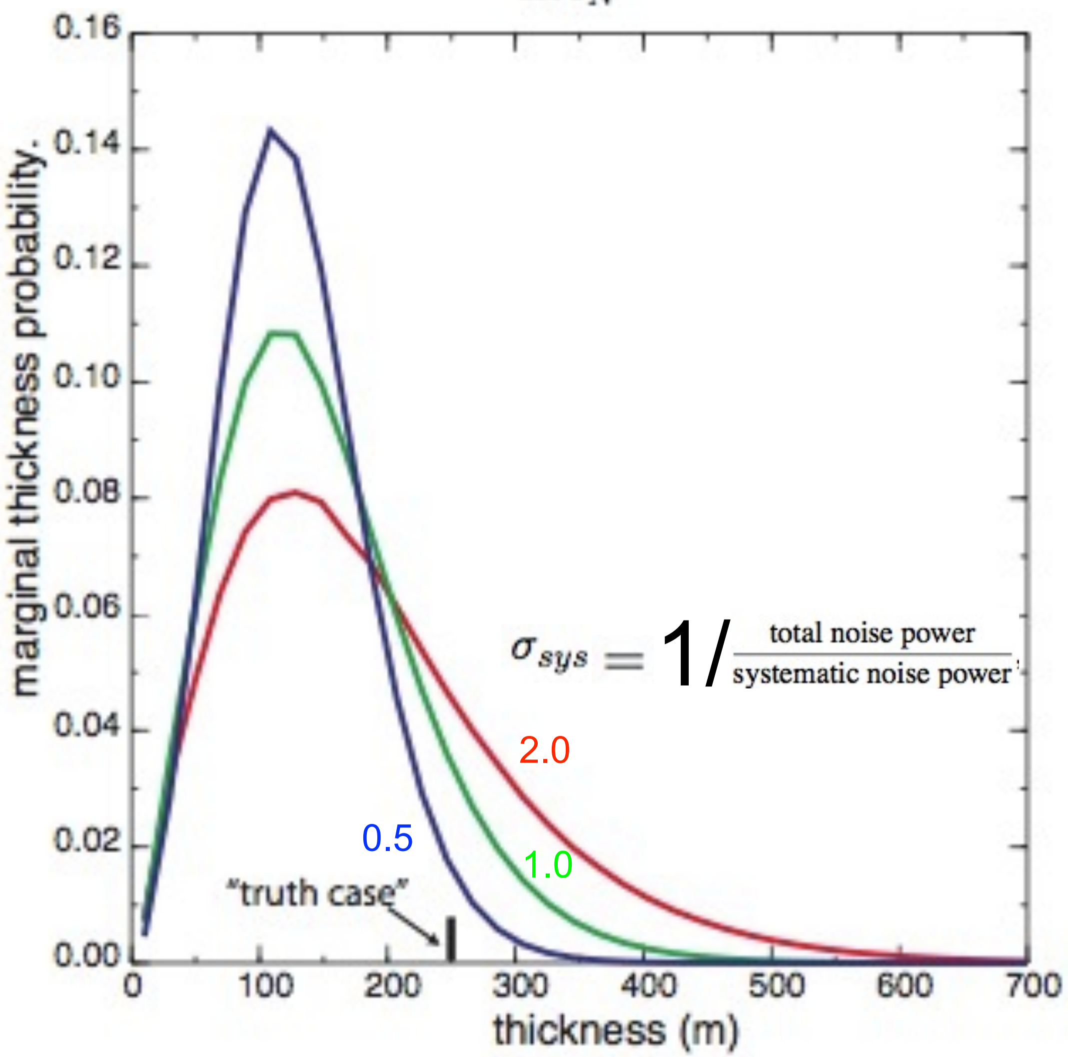
Classification of individual realizations



Corrections for systematic noise

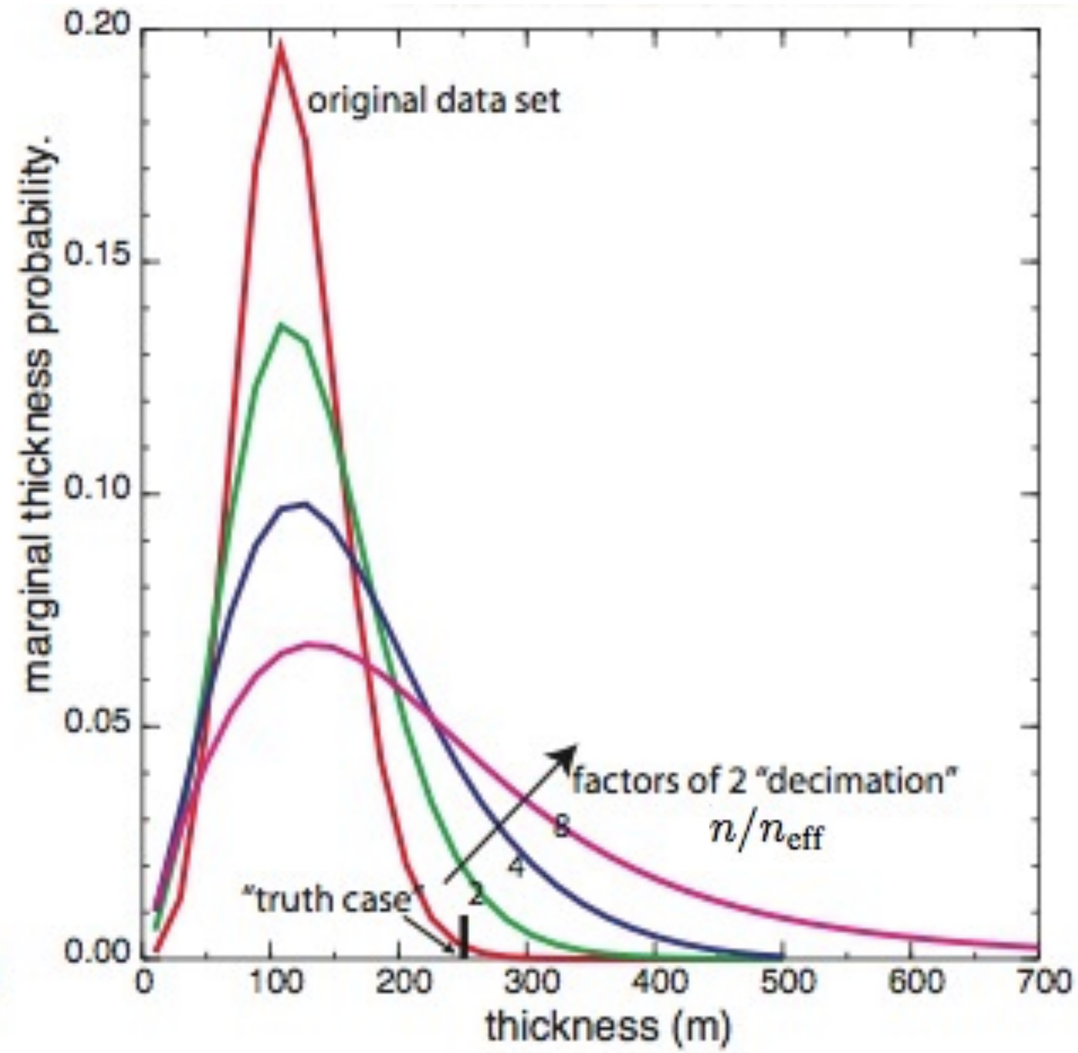
noise model fit

$$\mathbf{d} = \mathbf{F}(\mathbf{m}) + \underbrace{C_d^{1/2} X}_{\equiv X_N} .\mathbf{m}_n + \epsilon$$



data decimation

$$n_{\text{eff}} \approx \min\left(\frac{\text{total noise power}}{\text{systematic noise power}}, n\right)$$



Past track record & future

- Peer reviewed technology for probabilistic seismic and CSEM inversions
  - DELIVERY, http://tinyurl.com/ydqk7nc and http://tinyurl.com/ye2ld4f
  - deliveryCSEM, http://tinyurl.com/yl79g3g
- Seismic inversion well benchmarked on synthetic models and applied to over 25 assets by multiple companies
  - results cross validated to wells
  - pre-drill predictions verified by outcomes of drilling
- CSEM inversion well benchmarked on synthetic models and applied to several real datasets including the Cerah prospect in Block N (Sabah)
- Future
  - refinement and application of 1D method
  - extension into 2.5D using finite element methods with automatic grid refinement
  - development of simultaneous CSEM and seismic timelapse inversion

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P.S.: What I am really working on

- Understanding and prediction of self organisation of geologic sedimentation
- Developing and using a new renormalization theory, with linear convergent Wick expansion in terms of complexity of interaction
- Next seminar "Invariant actions -- dynamic DNA"

Collaborations and personnel into the future

- OCE Science Leader
  - Michael Glinsky (planned 2 month residence at Santa Fe Institute, Pawsey Supercomputer Centre Steering Committee and leading UQ for Resources Grand Challenge)
- OCE postdocs
  - Karen Livesey, theoretical physicist from UWA
  - Bela Nagy, mathematics from UBC and Santa Fe Institute
- PhD students
  - Zac Borden, computer simulation from UCSB
  - Youssef Mroueh, mathematics from L'Ecole Polytechnique
- Honours student
  - from UWA associated with my Adjunct Professorship in Physics
- Visitors & collaborators
  - Moshe Strauss, theoretical physicist from Israeli National Lab
  - Vivek Sarkar, computational science from Rice University
  - Henry Abarbanel, theoretical physicist from UCSD
  - Stephane Mallat, mathematics from L'Ecole Polytechnique
  - Tarabay Antoun, computer simulation from Lawrence Livermore National Lab
- Industrial application
  - Chevron SEED project
  - Woodside 10/11 budgeted technology project

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