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Guiding Center Atoms

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[M. E. Glinsky and T. M. O'Neil, Bul. Am. Phys. Soc. **33**, 1899 (1988)]

Abstract*

We say that a bound electron proton system is a guiding center atom, when the electron motion is classical and can be treated with guiding center drift theory; here, the system is assumed to reside in a large magnetic field. The electron motion is characterized by three well separated frequencies. The highest is the cyclotron frequency, the next is the bounce frequency of the electron along a field line in the Coulomb potential well, and the smallest is the ExB drift rotation frequency of the electron around the proton. In picturing the motion, one must realize that the cyclotron radius is much smaller than the distance between the electron and proton. Such atoms are expected to occur during the initial phase of recombination when a proton is introduced into a strongly magnetized and cryogenic pure electron plasma. Such atoms and the three particle recombination process leading to them are discussed. Also, we note that the anti-matter analogue of this process is of interest for current attempts to produce antihydrogen.[1]

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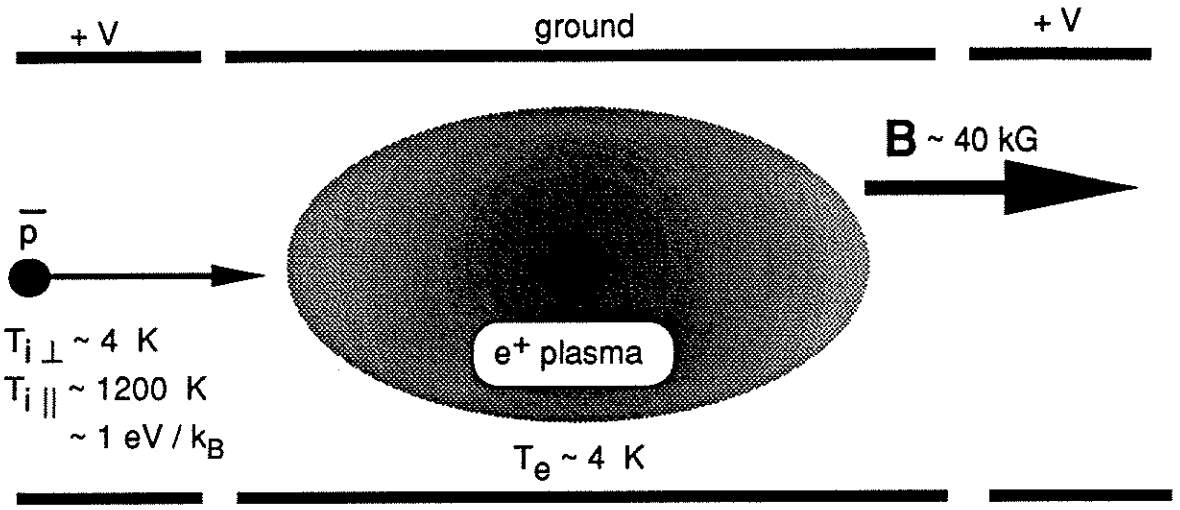
[1]G. Gabrielse, S. L. Rolston, L. Haarsma and W. Kells, Phys. Lett. **A129**, 38 (1988).

Motivation



How fast and by what mechanism will an antiproton and a positron recombine into antihydrogen?

References

- **Basic Trap Geometry**

  J.H. Malmberg, T. M. O'Neil, A.W. Hyatt and C.F. Driscoll, "The Cryogenic Pure Electron Plasma", *Proc. of 1984 Sendai Symposium on Plasma Nonlinear Phenomena*, 31 (1984).

- **Positron Trap**

  C. Surko et. al., Rev. Sci. Instr. **57** , 1862 (1986).

- **Antiproton Trap**

  G. Gabrielse et. al., Phys. Rev. Lett. **57**, 2504 (1986).

  X. Fei, R. Davisson and G. Gabrielse, Rev. Sci. Instr. **58**, 2197 (1987).

- **Antihydrogen Production**

  G. Gabrielse, S. L. Rolston, L. Haarsma and W. Kells, Phys. Lett. **A129**, 38 (1988).

- **Spectroscopy and Gravitation Study of Trapped Antihydrogen**

  G. Gabrielse, "Trapped Antihydrogen for Spectroscopy and Gravitation Studies: Is It Possible?", *Proc. of the Symposium on the Production and Investigation of Atomic Antimatter* , edited by H. Poth and A. Wolf (Scientific, Basel, 1988).

Parameter Regime

$$n b^3 \ll 1$$

(weakly correlated)

$$\frac{r_c}{b} \ll 1$$

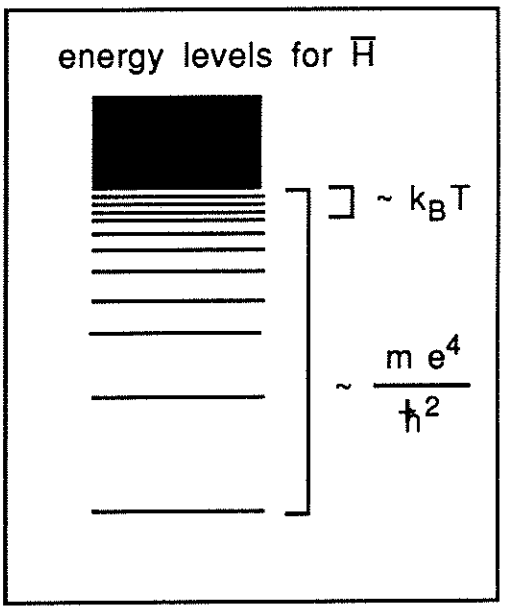
(guiding center dynamics)

$$\frac{\bar{v}_{i\perp}}{\bar{v}_e} \ll \frac{r_c}{b}$$

(small $\perp$ ion velocity)

$$\lambda_t \ll b \quad \text{or} \quad k_B T \ll \frac{m e^4}{2\hbar^2} \approx 13\ \text{eV}$$

(classical dynamics)



where

$b = \frac{e^2}{k_B T}$ = distance of closest approach

$r_c = \frac{\bar{v}_e}{\Omega_c}$ = cyclotron radius

$\Omega_c = \frac{e B}{m_e c}$ = cyclotron frequency

$\lambda_t = \frac{h}{m \bar{v}_e}$ = thermal de Broglie wavelength

Collisional 3-Body Recombination Dominates at Low Temperatures

$$\text{Rate}_{\text{3-body recombination}} \sim \underbrace{\left(n \bar{v}_e b^2\right)}_{\text{rate of collision}} \cdot \underbrace{\left(n b^3\right)}_{\substack{\text{probability another positron close enough to carry away momentum}}}$$

$$\sim \omega_{pe} \left(n b^3\right)^{3/2}$$

$$\sim \omega_{pe} \left(\frac{n^{1/3} e^2}{k_B T}\right)^{9/2}$$

At low temperatures the temperature scaling of this rate ($T^{-9/2}$) makes 3-body recombination the dominant process.

Monte Carlo Calculation of Rates

For B=0,
the calculation by Mansbach and Keck shows

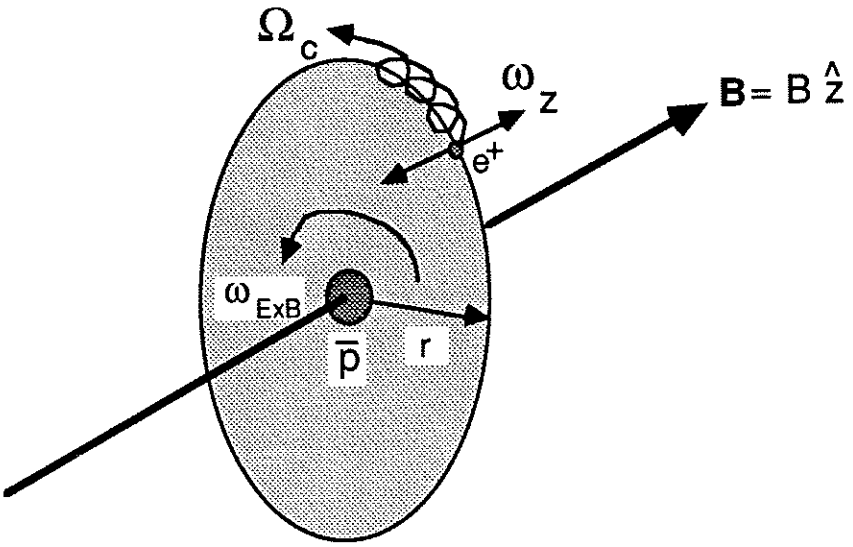
$$\text{Rate} \approx 0.76(4)\ n^2 b^5 \bar{v}_e$$

[P. Mansbach and J. Keck, Phys. Rev. **181**, 275 (1969).]
[B. Makin and P. Mansbach, Phys. Rev. Lett. **11**, 281 (1963).]

For $r_c << b$,
we have calculated

$$\text{Rate} \approx 0.070(10)\ n^2 b^5 \bar{v}_e$$

Guiding Center Dynamics



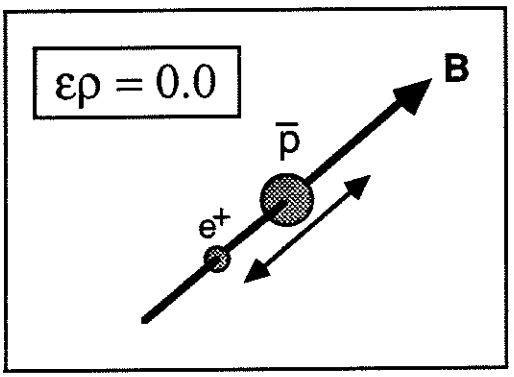
Ordering of Frequencies

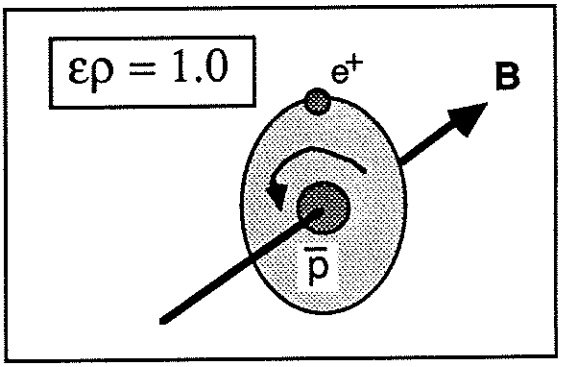
$$\Omega_c >> \omega_z >> \omega_{ExB} >> \frac{\bar{v}_{i\perp}}{r}$$

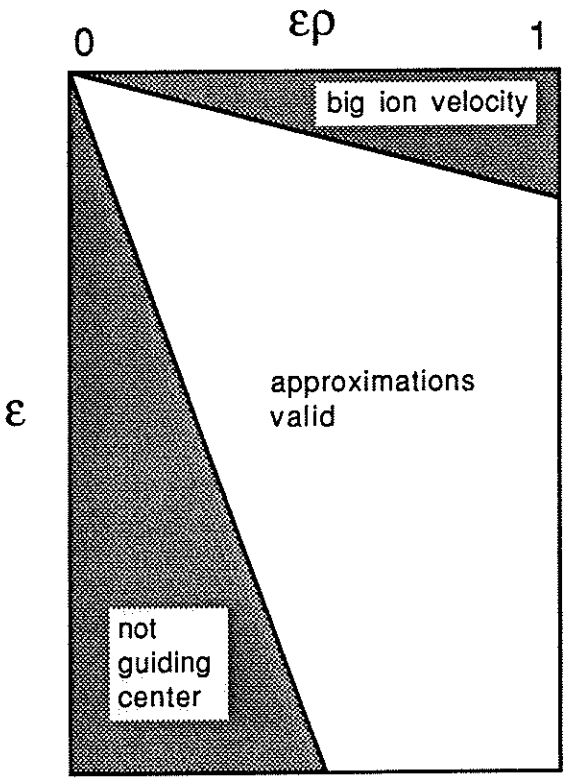
State of Guiding Center Atom

Given by $(\varepsilon,\varepsilon\rho)$ where

$$\varepsilon \equiv \frac{-\left(\frac{p_z^{\,2}}{2\,m_e} - \frac{e^2}{\sqrt{z^2+r^2}}\right)}{k_B T} \qquad \varepsilon\rho \equiv \varepsilon\,\frac{r}{b}$$







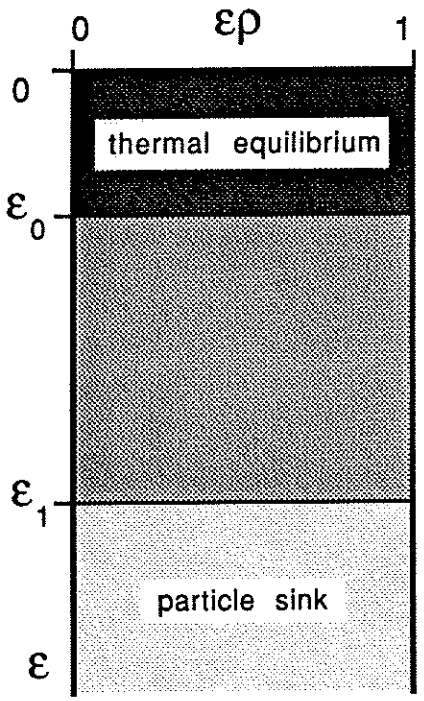
$$\varepsilon\rho > \varepsilon\left(\frac{r_c}{b}\right)^{2/3}$$

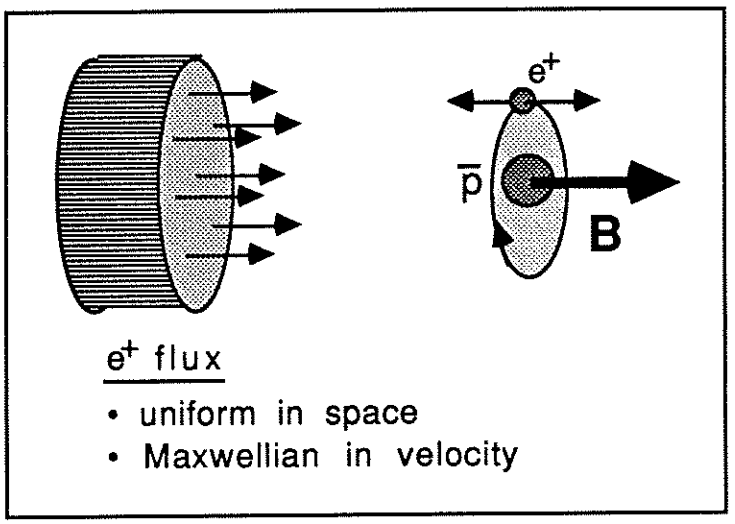
(guiding center dynamics)

$$\varepsilon\rho < \varepsilon\left[\left(\frac{\bar{v}_e}{\bar{v}_{i\perp}}\right)\left(\frac{r_c}{b}\right)\right]^{1/2}$$

(small $\perp$ ion velocity)

Monte Carlo Code





Numerical algorithm is:

• Pick state of guiding center atom in thermal distribution. Collide a positron from Maxwellian flux. Repeat these steps until an atom with energy below ε_0 is formed.

• Once formed, continue to collide positrons with the guiding center atom. This will trace out a trajectory in $(\varepsilon,\varepsilon\rho)$ space. Do this until the state of the atom crosses ε_0 or ε_1.

The recombination rate will be

$$\text{Rate}_{\text{recombine}} = \frac{1}{\left\langle \text{time for formation of atom which crosses } \varepsilon_1 \right\rangle}$$

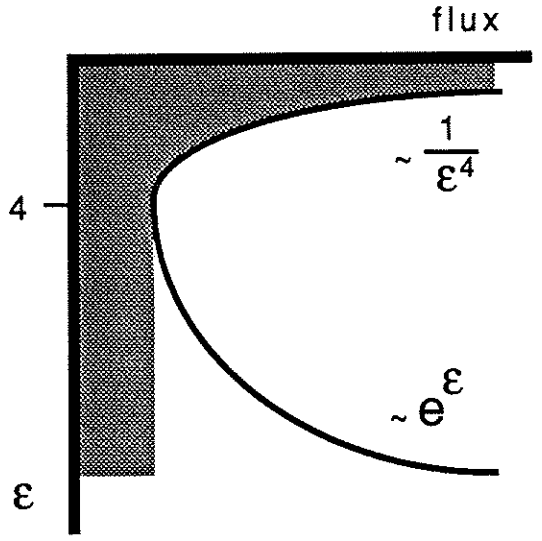
and the steady state distribution, $f_{ss}(\varepsilon,\varepsilon\rho)$, in the area between ε_0 and ε_1 will be proportional to the amount of time spent in a $\Delta\varepsilon\Delta\varepsilon\rho$ box at $(\varepsilon,\varepsilon\rho)$ divided by $\Delta\varepsilon\Delta\varepsilon\rho$.

Bottleneck

The one way thermal equilibrium flux through an ε=constant surface scales as

$$\text{flux} \sim \frac{e^{\varepsilon}}{\varepsilon^{4}}$$

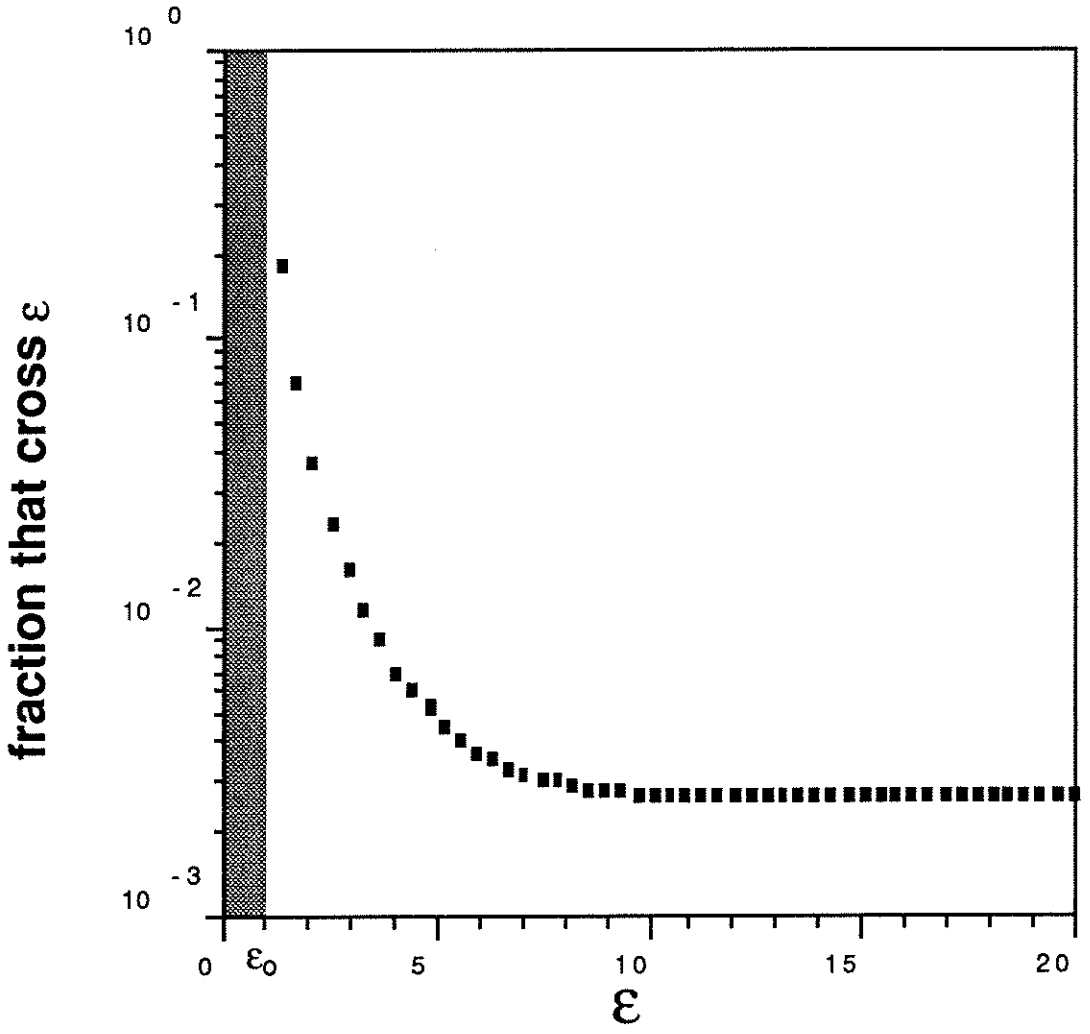
dominated by the phase space factor, $1/\varepsilon^{3.5}$, and the Boltzmann factor, e^{ε}. This flux has a strong minimum at ε=4.



In steady state, the rate through any ε surface will be a constant. It is obvious that this rate can not be larger than the minimum flux through an ε=constant surface. Therefore, the distribution function for states above the bottleneck will nearly attain its thermal equilibrium value and below the flux minimum the distribution function will be significantly less than its equilibrium value. This is in close analogy with water flowing through the neck of a bottle, hence the name bottleneck.

Bottleneck Exists

Fraction of Atoms Initially Formed (ε_o =1) that Make it to States with Energy > ε .

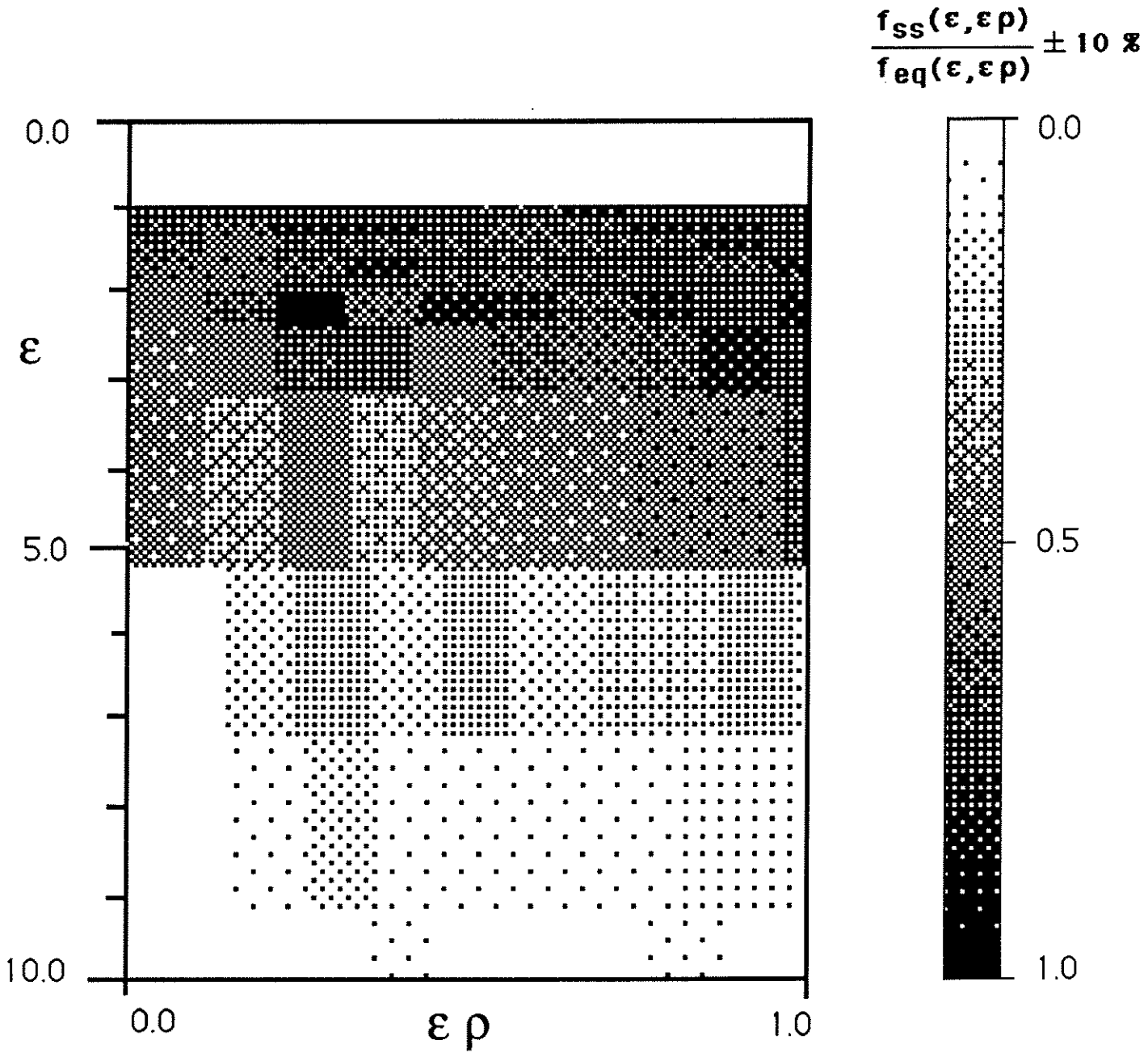


If a bound positron makes it to a state below the bottleneck of $\varepsilon \approx 5$, then it continues to be more bound. This makes it possible to unambiguously define a recombination. The rate for such recombinations is

$$\text{Rate}_{\text{Recombination}} = 0.070(10)\ n^2 b^5 \bar{v}_e$$

Steady State Distribution

(normalized to thermal equilibrium)



Time Scales

There are three time scales which are well separated if $nb^3 << 1$. The first is the time scale over which a colliding positron interacts with a guiding center atom,

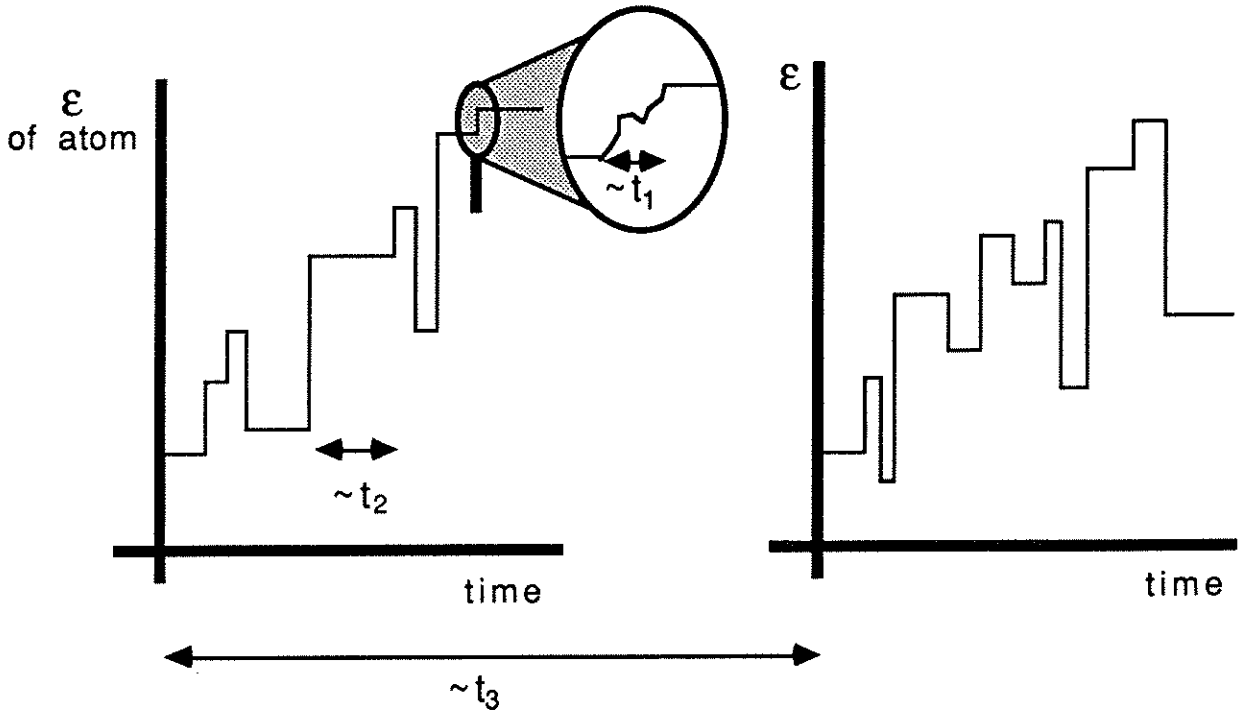
$$t_1 = \text{duration of collision} \sim \left(\frac{\bar{v}_e}{b}\right)^{-1}$$

The second is the time scale one must wait until a positron collides within an impact parameter, b, of the guiding center atom,

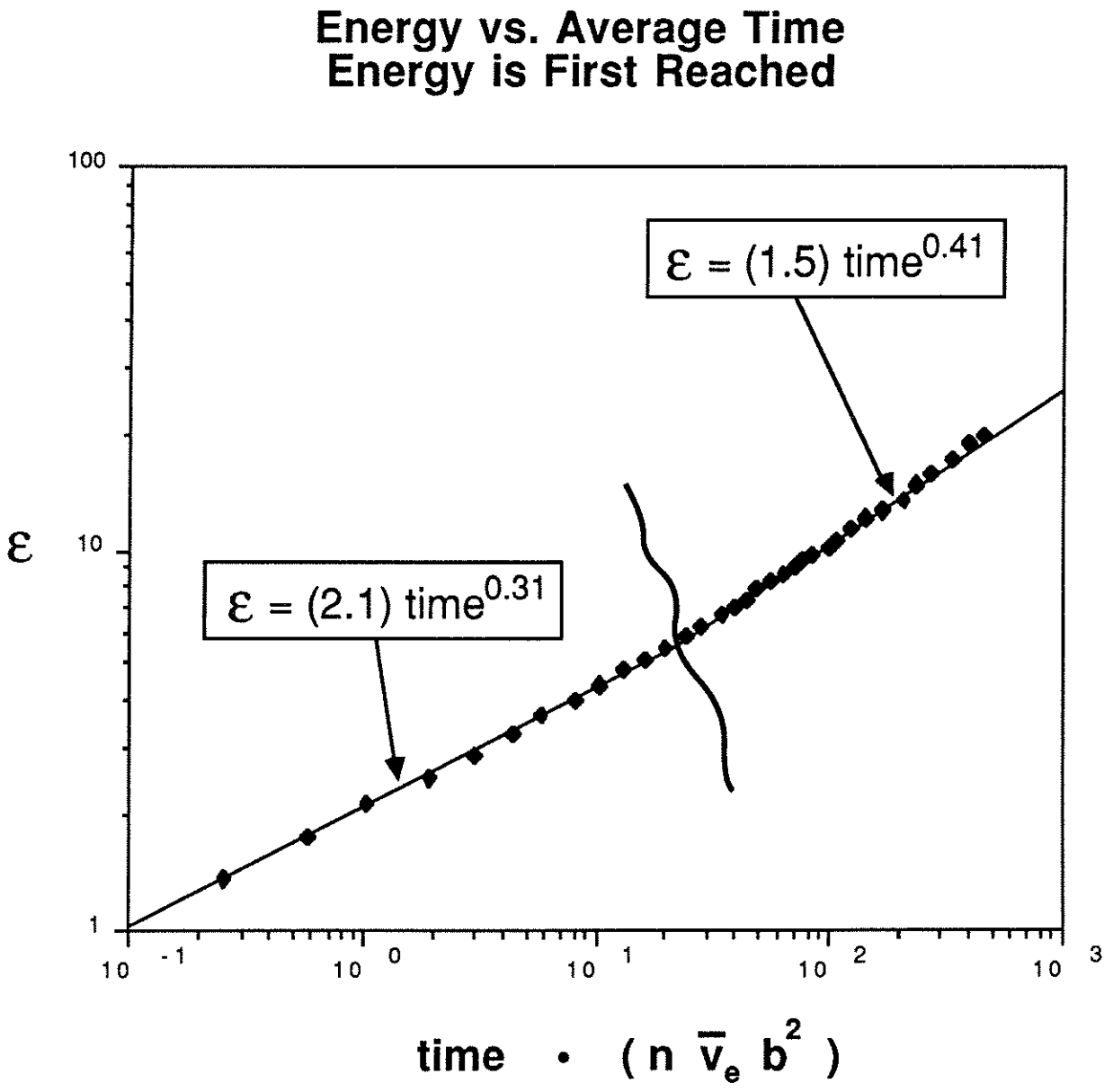
$$t_2 = \text{time between collision} \sim \left(n\, b^2\, \bar{v}_e\right)^{-1} \sim t_1 \left(n\, b^3\right)^{-1}$$

The third is the time scale for an antihydrogen atom to be formed,

$$t_3 = \text{recombination time} \sim \left(n^2\, b^5\, \bar{v}_e\right)^{-1} \sim t_2 \left(n\, b^3\right)^{-1}$$



Time to Capture Positron



Scaling of Time to Capture

Assume that the rate of collision of a positron with the atom is proportional to the area within an adiabatic cutoff

$$\text{Rate} \sim r_{\text{cutoff}}^2$$

The cutoff for small ε is the radius for which

$$\frac{r_{\text{cutoff}}}{v_e}\omega_z \sim 1 \quad \Rightarrow \quad \frac{r_{\text{cutoff}}}{b} \sim \frac{1}{\varepsilon^{3/2}}$$

For large ε the above cutoff gets much smaller than the average separation of the positron from the antiproton,

$$\frac{< r >}{b} \sim \frac{1}{\varepsilon}$$

Therefore, for large ε, $< r >$ must be used in place of r_{cutoff} to determine the rate.

From the numerics the scaling of the average step size has been found to be

$$< \Delta\varepsilon > \sim \varepsilon$$

Now

$$\frac{d\varepsilon}{dt} = \frac{\Delta\varepsilon}{\Delta t} = (\text{Rate})\,(\Delta\varepsilon) \sim \frac{1}{\varepsilon^2} \quad \text{(small } \varepsilon\text{)}$$

$$\sim \frac{1}{\varepsilon} \quad \text{(large } \varepsilon\text{)}$$

Whose solution is

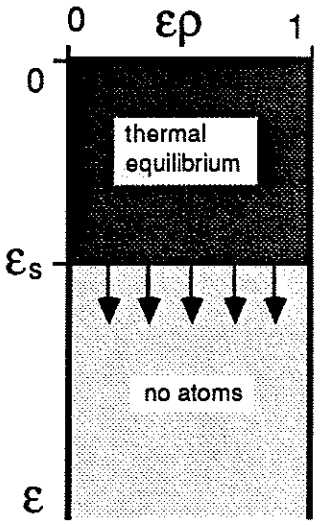
$$\varepsilon(t) \sim t^{1/3} \quad \text{(small } \varepsilon\text{)}$$

$$\sim t^{1/2} \quad \text{(large } \varepsilon\text{)}$$

Calculation of One Way Flux for Thermal Equilibrium

$R_s(\varepsilon_s) \equiv$ flux through surface, S, specified by $(\varepsilon = \varepsilon_s)$

$$= \int_{S'} e^{\frac{-H}{k_B T}} \hat{n} \cdot \vec{v}\, dS$$



where the following guiding center Hamiltonian is used

$$H = \frac{p_{1z}^2}{2m} + \frac{p_{2z}^2}{2m} - \frac{e^2}{\sqrt{z_1^2 + x_1^2 + \left(\frac{p_{1x}}{m\Omega_c}\right)^2}} - \frac{e^2}{\sqrt{z_2^2 + x_2^2 + \left(\frac{p_{2x}}{m\Omega_c}\right)^2}} + \frac{e^2}{\sqrt{(z_1 - z_2)^2 + (x_1 - x_2)^2 + \left(\frac{p_{1x} - p_{2x}}{m\Omega_c}\right)^2}}$$

$R_S(\varepsilon_S)$ is divergent unless the following adiabatic cutoff for positron guiding center atom collision is imposed

$$\frac{r_{cutoff}}{b} = \frac{\rho_o}{\varepsilon}$$

then

$$R_s(\varepsilon_s) \approx \left(n^2 b^5 \bar{v}_e\right)\left(\frac{(2\pi)^{3/2}}{3}\right)\left(\frac{e^\varepsilon}{\varepsilon^4}\right)\left[\underset{\parallel \text{ flux}}{\rho_o} + \underset{\substack{\perp \text{ flux} \\ (r < r_{cutoff})}}{O\left(\frac{r_c}{b}\right)} + \underset{\substack{\perp \text{ flux} \\ (r > r_{cutoff})}}{O\left(\left(\frac{r_c}{b}\right)^2 \ln\left(\frac{b}{r_c}\right)\right)}\right]$$

which has a minimum value of

$$\left(R_s\right)_{min} \approx (1.1)\left(n^2 b^5 \bar{v}_e\right)\rho_o$$

Comparison of Monte Carlo Code's Equilibrium One Way Flux to the Analytically Calculated Flux.