

Appendix D Formula for R_s with Separatrix

$$\bar{r}_{s1} \equiv \left[\left(\frac{m_e}{m_i} \right)^{1/2} \left(\frac{b}{r_{ce}} \right) \right]^{-2/3} = \left(\frac{1}{a} \right)^{1/3}$$

$$\begin{aligned} \bar{r}_{s2} &\equiv \left[\frac{v_i}{v_e} \frac{b}{r_{ce}} \right]^{-1/2} = \left(\frac{T_i}{T_e} \right)^{-1/4} \left[\left(\frac{m_e}{m_i} \right)^{1/2} \left(\frac{b}{r_{ce}} \right) \right]^{-1/2} \\ &= \left(\frac{R}{a} \right)^{-1/2} = \left(\frac{T_e}{T_i} \right)^{1/4} \bar{r}_{s1}^{3/4} \end{aligned}$$

$$\begin{aligned} \bar{r}_{ci} &\equiv \frac{r_{ci}}{b} = \frac{r_{ce}}{b} \frac{v_i}{v_e} \frac{m_i}{m_e} = \frac{T_i}{T_e} \frac{v_e}{v_i} \frac{r_{ce}}{b} \\ &= \left(\frac{v_i}{v_e} \frac{b}{r_{ce}} \right)^{-1} \frac{T_i}{T_e} = \bar{r}_{s2}^2 \frac{T_i}{T_e} \\ &= \left(\frac{T_e}{T_i} \right)^{1/2} \bar{r}_{s1}^{3/2} \left(\frac{T_i}{T_e} \right) \\ &= \left(\frac{T_i}{T_e} \right)^{1/2} \bar{r}_{s1}^{3/2} \end{aligned}$$

$$R_3 \sim n^2 \bar{v} b^5 \max \left[1, e^{1/r_{ci} \bar{r}_{s2}^{a/2}} \right]$$

$$\begin{aligned} \frac{T_i}{T_e} &\equiv \xi \quad \eta \equiv \bar{r}_{s1} = \left(\frac{m_e}{m_i} \right)^{-1/3} \left(\frac{b}{r_{ce}} \right)^{-2/3} \\ &= \left(\frac{1}{a} \right) \left(\frac{R}{a} \right)^2 = \left(\frac{m_i}{m_e} \right)^{1/3} \left(\frac{v_{ce}}{v} \right)^{2/3} \end{aligned}$$

$$\begin{aligned} 1/\bar{r}_{ci} &= \xi^{-1/2} \eta^{-3/2} \\ \bar{r}_{s2}^{a/2} &= \xi^{(a/2)(-1/4)} \eta^{(a/2)(3/4)} = \left(\frac{\omega_D}{\Omega_i} \right)^{1/3} \\ &= \left[\left(\frac{m_i}{m_e} \right) \left(\frac{r_{ce}}{b} \right)^2 \right]^{1/3} \\ &= \xi^{-9/8} \eta^{27/8} \end{aligned}$$

$$R_3 \sim n^2 \bar{v} b^5 \max \left[1, e^{\xi^{-1/2} \eta^{-3/2} \xi^{-9/8} \eta^{27/8}} \right]$$

Point where $\bar{R}_3$ returns to 1 ($\eta < 1$)

$$\bar{r}_{s1} \rightarrow \left(\frac{T_i}{T_e} \right)^{-1/3} \left[\frac{q}{4} \ln \frac{T_i}{T_e} \right]^{-2/3} = \xi^{-1/3} \left[\frac{q}{4} \ln \xi \right]^{-2/3}$$

$$\text{as } T_i/T_e \rightarrow \infty$$

$$\text{where } \bar{R}_3 \sim 1$$

$$e^{1/\bar{r}_{ci}} \bar{r}_{s2}^{q/2} = 1$$

$$\frac{1}{\bar{r}_{ci}} + \frac{q}{2} \ln \bar{r}_{s2} = 0$$

$$\xi^{-1/2} \eta^{-3/2} + \frac{q}{2} \ln \left[\xi^{-1/4} \eta^{3/4} \right] = 0$$

$$\xi^{-1/2} \left(\xi^{1/4} \eta^{3/4} \right) \left(\xi^{-1/2} \eta^{-3/2} \right)^{-1/2}$$

$$\text{define } y \equiv \xi^{-1/2} \eta^{-3/2} = \frac{1}{\bar{r}_{ci}}$$

$$y + \frac{q}{2} \ln \left(\xi^{-1/2} y^{-1/2} \right) = 0$$

$$y + \frac{q}{2} \left[-\frac{1}{2} \ln \xi - \frac{1}{2} \ln y \right] = 0$$

$$\boxed{y - \frac{q}{4} [\ln \xi + \ln y] = 0} \text{ solve for } y$$

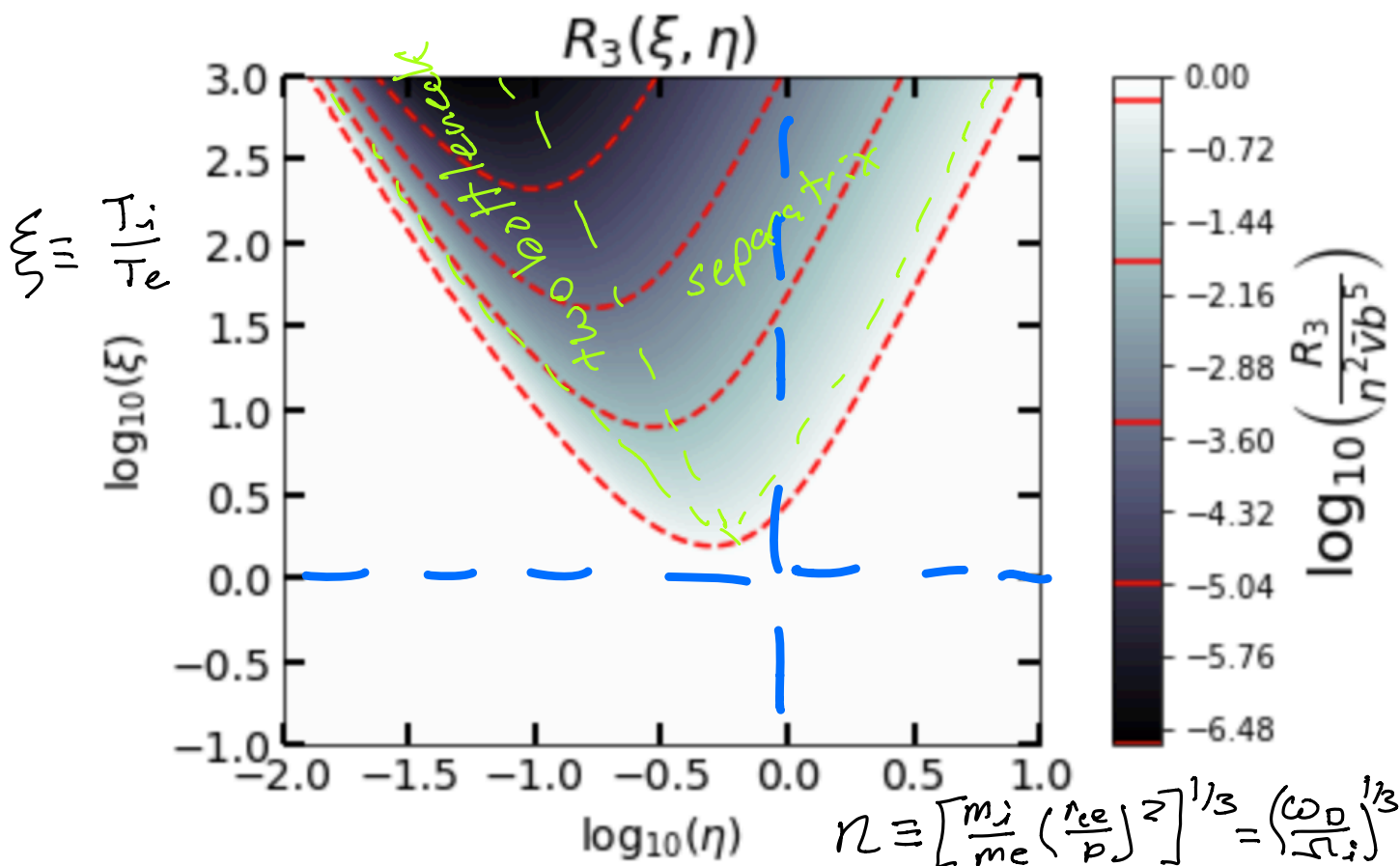
$$\text{as } \xi \rightarrow \infty \quad y \rightarrow \frac{q}{4} \ln \xi$$

$$y = \frac{q}{4} (\ln \xi + \ln y)$$

$$\xi^{-1/2} \eta^{-3/2} = \frac{q}{4} \ln \xi$$

$$\eta = \left[\frac{q}{4} \xi^{1/2} \ln \xi \right]^{-2/3}$$

$$= \xi^{-1/3} \left[\frac{q}{4} \ln \xi \right]^{-2/3}$$



two bottleneck

separatrix

